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HYDROFOIL SHIP
CONSTANT LIFT CONTROL SYSTEM
PART I: HYDRODYNAMICS

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DEPARTMENT OF
NAVAL ARCHITECTURE
AND MARINE ENGINEERING M.I.T.

June, 1969

HYDROFOIL SHIP CONSTANT LIFT CONTROL SYSTEM PART I: HYDRODYNAMICS

by

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S.B., Massachusetts Institute of Technology
(1962)

Submitted in Partial Fulfillment
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ABSTRACT

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CONSTANT LIFT CONTROL SYSTEM
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MICHAEL ROY TERRY

Submitted to the Departments of
NAVAL ARCHITECTURE and MARINE ENGINEERING
and MECHANICAL ENGINEERING on
23 May 1969, in partial
fulfillment of the requirements
for the Degrees of NAVAL ENGINEER,
and MASTER of SCIENCE in MECHANICAL ENGINEERING.

This paper analyzes the unsteady hydrodynamics of 1) FULL INCIDENCE; 2) TRAILING EDGE FLAP; and 3) TAB FOIL hydrodynamic control devices. The device is to be used in a constant lift, wave alleviation control system on an OPEN-OCEAN HYDROFOIL SHIP. Each device is compared in POWER, DRAG, and WEIGHT. The conclusions reached are:

1. Wave Alleviation is an important aspect of hydrofoil ship control system design.

2. Unsteady Analysis is vital to understanding the response of a foil in waves.

3. Negative Control System Power is required for wave alleviation by a tab foil with a lagging tab.

4. Trailing Edge Flap requires less overall system power than a Full Incidence Foil.

5. Hydrofoil Ship Operation Policy will be greatly affected by unsteady hydrodynamics.

6. Unsteady Foil Response is not a unique function of encounter frequency.

Thesis Supervisor: Damon Ellis Cummings
Title: Assistant Professor of Naval Architecture

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The computer programs were executed with dispatch by the Information Processing Center on their IBM 360, using a Fortran IV G Compiler, and a Calcomp 565 Plotter.

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SYMBOLS

a = NACA camber loading parameter

a = α hinge point in % of b

A = Foil area

b = Semichord

c = Chord

c = β hinge point in % of b

C = Designates circulatory

C_l = Lift coefficient $L/((1/2)\rho V^2 A)$

C_{l_0} = Lift coefficient at ideal angle of attack

$\overline{C(k)}$ = Theodorsen 2-D unsteady lift function

d = Source position

D = Drag

e = Designates encounter

g = Gravitation constant ft^2/sec

h = Keel to foilborne waterline height

i = $\sqrt{-1}$

I_a = Main foil inertia/ft span

I_b = Flap inertia/ft span

$J_n(k)$ = Bessel Function of order n

k = Reduced frequency

L = Lift/ft span

L = Length

m = Source strength/ft span

M = Moment/ft span

M = Mass of the ship

NC = Denotes non-circulatory

Q = Flow rate

s = Span

t = Time

t = Thickness

T = Period

U = Ship Speed

V = Wave Phase Velocity

w = Denotes wave

x = Distance in horizontal direction

x_0 = Longitudinal distance from reference point to center of gravity

y = Distance in vertical direction

z = Distance traveled in vertical direction

$\dot{z}$ = Vertical velocity

$\ddot{z}$ = Vertical acceleration

Z = Vertical force

α = Pitch angle of incidence about ba

α_0 = Pitch angle of incidence magnitude

$\dot{\alpha}$ = Incidence angular velocity

$\ddot{\alpha}$ = Incidence angular acceleration

β = Pitch angle of flap about bc

β_0 = Pitch angle of flap magnitude

$\dot{\beta}$ = Flap angular velocity

$\ddot{\beta}$ = Flap angular acceleration

Δ = Ship displacement

Δ = Increment of change

ε = Small increment

η_p = Propulsive coefficient

η = Wave height

η_0 = Wave height magnitude

$\dot{\eta}$ = Wave orbital velocity

$\ddot{\eta}$ = Wave orbital acceleration

$\ddot{\theta}$ = Ship pitch acceleration

λ = Wave length

π = 3.1416...

ρ = Fluid density in slugs/ft^3

ϕ = phase angle

ϕ_1 = $\angle \alpha$

ϕ_W = $\angle \dot{\eta}$

$\Delta\phi$ = Phase angle between α, β

ψ = Angle of attack of ship to waves

ψ = Stream function

ω = Circular frequency radians/sec

MAJOR COMPUTER PROGRAM SYMBOLS

A	=	Plotting symbol for $\lvert\alpha\rvert$
$\overline{\text{ALPHA}}$	=	α
$\overline{\text{ALPHADOT}}$	=	$\dot{\alpha}$
$\overline{\text{ALPHADDOT}}$	=	$\ddot{\alpha}$
B	=	b
B	=	Plotting symbol for $\lvert\beta\rvert$
BA	=	β_0/α_0
$\overline{\text{BETA}}$	=	β
$\overline{\text{BETADOT}}$	=	$\dot{\beta}$
$\overline{\text{BETADDOT}}$	=	$\ddot{\beta}$
C	=	c hinge line in % of b
CA	=	k
CK	=	$\overline{C(k)}$
DPHE	=	$\Delta\phi$
FIA	=	I_a
FIB	=	I_b
HW	=	η
$\overline{\text{LIFTW}}$	=	L_w
$\overline{\text{LIFTA}}$	=	L_α
$\overline{\text{LIFTB}}$	=	L_β
$\overline{\text{LIFTOT}}$	=	Total lift per unit span
$\overline{\text{MOMTW}}$	=	M_w about ba due to waves
$\overline{\text{MOMTBW}}$	=	M_w about bc due to waves
$\overline{\text{MOMTA}}$	=	M_α about ba due to α

MOMTBA = M_{α} about bc due to α

MOMTB = M_{β} about ba due to β

MOMTBB = M_{β} about bc due to β

MOMTOT = Total hydrodynamic moment about ba per unit span

MOMTBT = Total hydrodynamic moment about bc per unit span

MOMAPA = Required applied moment about ba per unit span

MOMAPB = Required applied moment about bc per unit span

P = Plotting symbol for PAVEA

PAVEA = Average power about ba per unit span

PAVEB = Average power about bc per unit span

PHEW = ϕ_w

Q = Plotting symbol for PAVEB

RO = ρ

SI = ψ

UPW = Upwash velocity $\dot{\eta}$

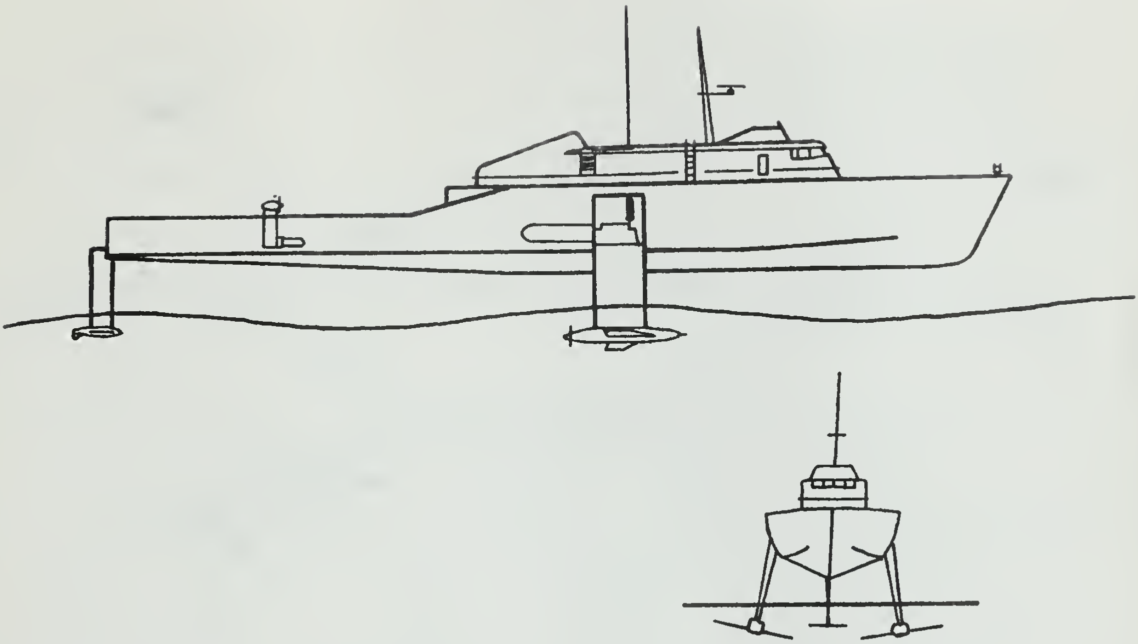
WE = ω_e

WL = λ

Y = Plotting symbol for $\angle\beta$

Z = Plotting symbol for $\angle\alpha$

A = a hinge line in % of b



I. INTRODUCTION

Renewed interest in the use of hydrofoil lifting surfaces to raise a ship above the surface of the water occurred in the 1950's. Since that time, development of complete hydrofoil ship systems has progressed at a rapid rate. During the recent development stages, the encounter between ships and open-ocean waves has identified the problem of ship motion control as being critical. The foil encounters lift changes caused by wave particle orbital motion. The wave particle orbital velocity is superposed on the uniform in-flow velocity (ship's speed through the water). The superposition results in an angle of attack change relative to the foil and, therefore, changes its lift. The changes in lift are disturbing forces to the ship and occur at the encounter frequency of the ship meeting the waves. Figure (1) depicts this situation.

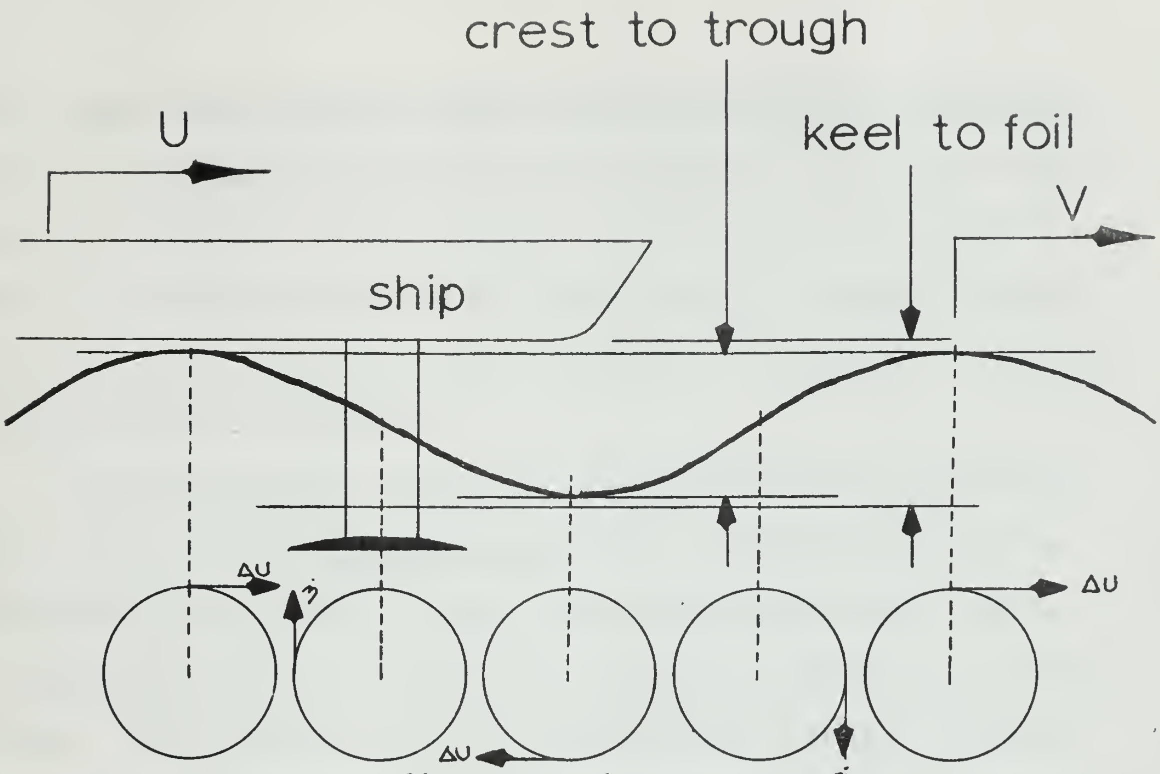


figure 1

<u>This paper will address only open-ocean waves and ocean-going hydrofoil ships.</u>

During the early stages of development, attention was focused on the choice of foil configuration for best results. From that lively discussion and experimentation, two major design configurations developed. The first is commonly referred to as <u>surface piercing</u>, and the second as <u>fully submerged</u>. Figure (2) depicts these configurations.

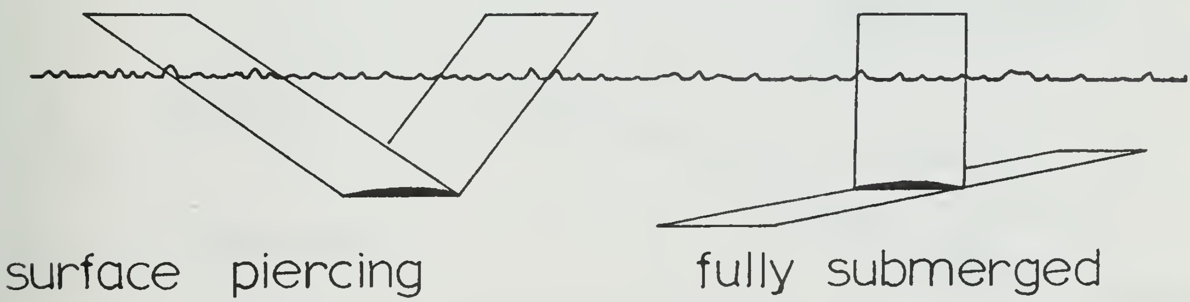


figure 2

This paper will address only the fully submerged configuration. Hydrofoil ships with fully submerged foils are inherently unstable in heave, pitch, and roll when the hull is out of the water, because the foil has very little sensitivity to depth below the water surface. Therefore, a control system must be provided.

While foilborne, two modes of operation are possible. The first is the platform mode where the ship's motion is maintained at constant height above the mean water level, at constant attitude about the height, and with no accelerations. The platform mode is only possible in wave heights that do not exceed the keel-to-foilborne waterline distance. The second is the contour mode where the ship's motion is maintained at constant height above the local water surface at two points along the ship's length. The contour mode is only necessary in wave heights that exceed the keel-to-foilborne waterline distance. Figure (3) depicts these modes.

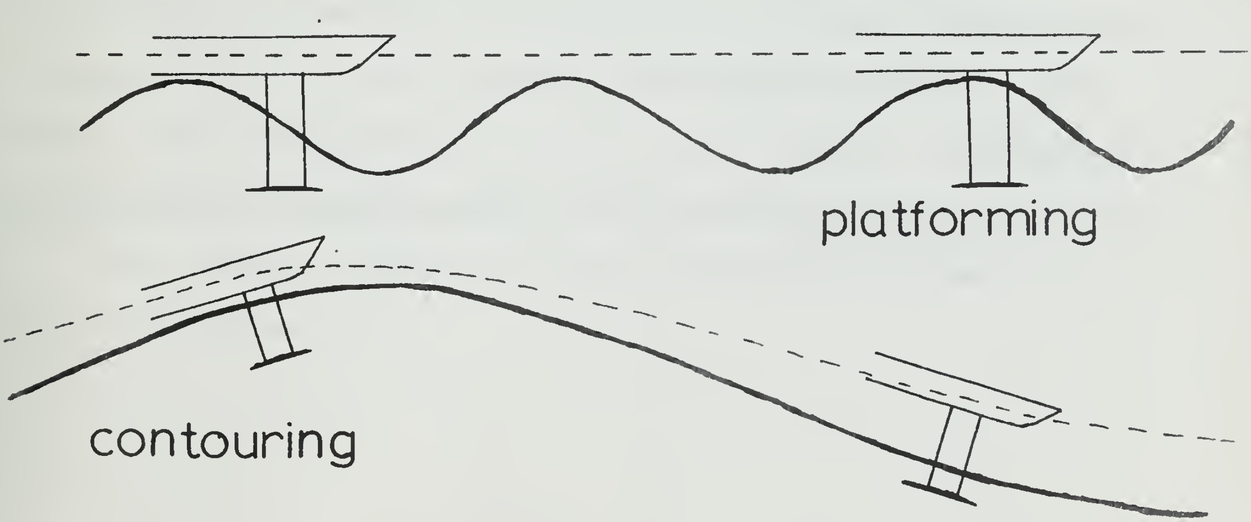


figure 3

This paper will address only the platform mode and the wave systems that allow the use of that mode.

The design of motion control for a hydrofoil ship must meet the three basic objectives outlined in Figure (4).

STABILITY	(Maintaining Commanded Equilibrium Attitude)
MANEUVERABILITY	(Changing to a New Commanded Equilibrium)
WAVE ALLEVIATION	(Reducing the Effect on Lift due to Waves)

figure 4

Most attention in the past has been concentrated on stability and maneuverability control functions. The effect of waves on the ship has been treated as a disturbance from equilibrium, and the stability elements of the control system have been sized to be able to correct the ship's attitude and reduce the accelerations to a tolerable level. In contrast, this paper will address only the wave alleviation objective.

The control system is usually composed of elements as shown in Figure (5).

SENSORS	(ELECTRONIC)
AUTOPILOT/COMPUTER	(ELECTRONIC)
SERVO	(FLUID POWER)
HYDRODYNAMIC DEVICE	(LIFTING SURFACE)

figure 5

<u>This paper will address only the Hydrodynamic Device.</u>

II. PROJECT

The objective of the above focus is now described. An open-ocean hydrofoil ship will operate, at times, when wave systems are present for the complete voyage. Operation in waves is, therefore, an important aspect of the design. Would the control system perform better than present day systems if the approach to its design considered only wave alleviation as the objective? The hypothesis is that the power level of the control system operating in waves could be significantly reduced if this approach was taken. Also, the wave alleviation control system could be integrated with a present day standard stability and maneuverability control system to produce a control system with overall performance superior to today's design. The stabilizing and maneuvering elements would effectively be operating as if they were in calm water, when in fact the hydrofoil ship is flying in open-ocean waves, Figure 6.

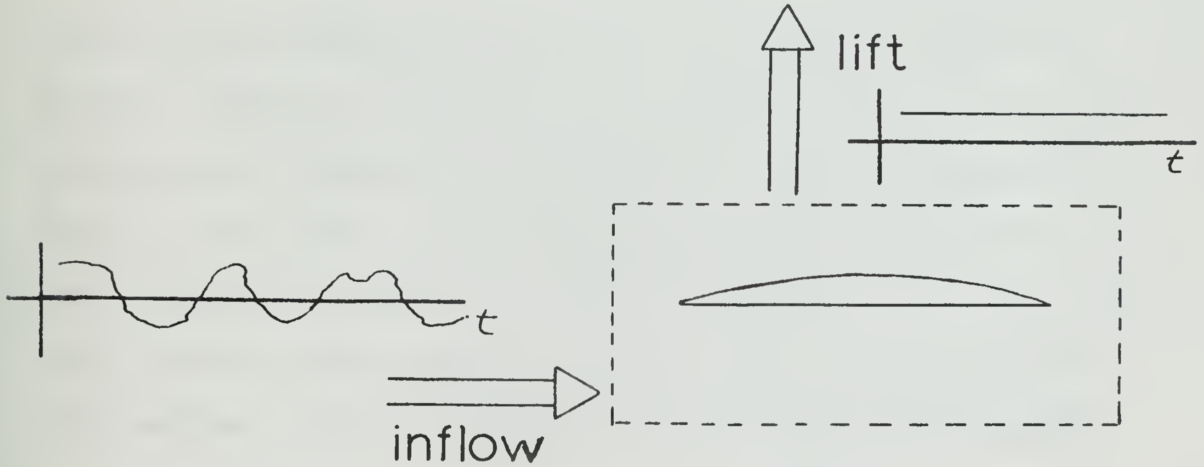


figure 6

In the process of designing the optimum control system for wave alleviation, different solutions must be tested. The criteria best suited for this test is minimum overall system power. The three elements that contribute directly to overall system power are shown in Figure (7).

POWER	Power to Operate the Control Surface
DRAG	Power to Overcome the Drag of the Hydrodynamic Device
WEIGHT	Power to Lift, Through the Lift/Drag Ratio, the Weight of the Control System

figure 7

For different designs, any one of the above elements of overall control system power may predominate. The task of designing the control system can be broken down into the design of the four basic elements--sensor, autopilot, servo, and hydrodynamic device as shown in Figure (5). "Hydrofoil Ship Constant-Lift Control System: Part I Hydrodynamics" will address only the fourth element--Hydrodynamic Force Producing Device.

In summary, Figure (8) shows the steps in focusing this thesis within the overall project.

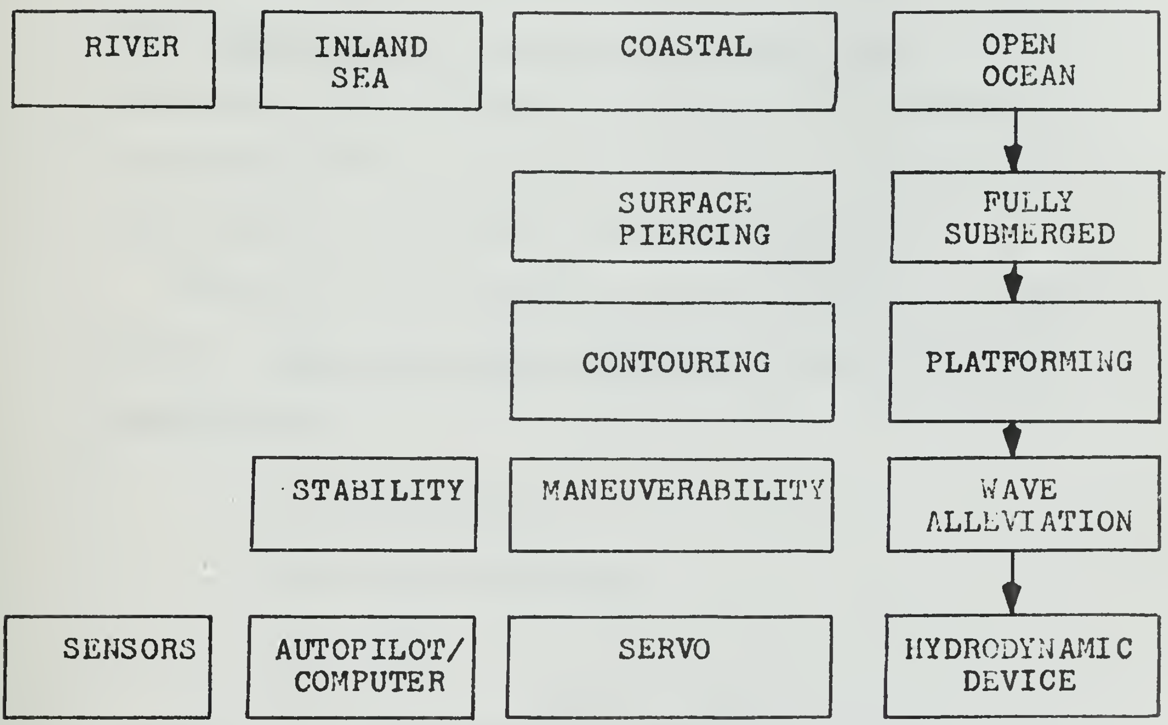


figure 8

III. HYDRODYNAMIC DEVICE

A. DEVICES

The first step in the plan to find the minimum power device is to develop as many candidates as possible for closer scrutiny. In general terms, the hydrodynamic device must effect a force. Although there are a number of ways of effecting a force, this paper will consider only those that develop lift. This is a reasonable specialization because the hydrofoil ship utilizes lifting surfaces for lifting itself out of the water and for stability and maneuverability.

Table (1) is an attempt at classifying the candidates. This paper will pursue only the following candidates:

1. Full Incidence
2. Trailing Edge Flap
3. Tab Foil

Appendix (A) describes the remaining candidates and discusses some of their characteristics in qualitative terms.

TABLE (1)

ANGLE OF ATTACK CHANGERS

Rotating Foil (Full Incidence)

Translating Foil (Suspended Foil)

CAMBER CHANGERS

Partial Chord Mechanical (Trailing Edge Flap)

Partial Chord Fluid (Jet Flap)

Upper Surface Mechanical (Membrane)

Upper Surface Fluid (Fluid Camber)

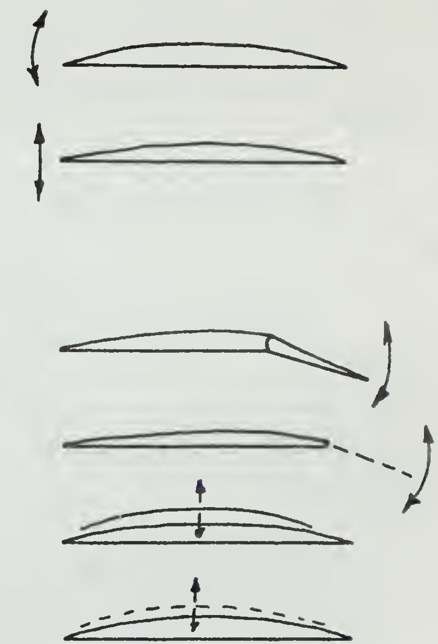


TABLE (1) (CONT'D)

CAMBER AND ANGLE OF ATTACK CHANGERS

Rotating Foil/Partial Chord Mechanical (Tab Foil)

Rotating Foil/Partial Chord Fluid (Jet Tab)

Translating Foil/Partial Chord Mechanical (Suspended Flap)

Translating Foil/Partial Chord Fluid (Suspended Jet Flap)

LIFT SPOILERS

Mechanical (Spoilers)

Fluid (Ventilation)

B. SHIP

Before proceeding with a quantitative analysis of the three candidates, a reference hydrofoil ship should be defined for use as a common denominator for each design. Table (2) lists the parameters of the reference hydrofoil ship chosen.

TABLE (2)

Displacement	Δ	= 300 Tons
Length Overall	L	= 220 Ft
Design Speed	U	= 40 kts
Design Lift/Drag	L/D	= 7
Design Propulsive Coefficient	η_p	= .6
Each of 2 Main Lifting Foils	NACA 16-408	
Thickness/Chord	t/c	= .08
Mean Line	a	= 1.0
Design Lift Coefficient	C_{l_0}	= .4
Chord	c	= 8.58 ft
Semichord	b	= 4.29 ft
Span	s	= 21.25 ft
Weight	wt.	= 7 tons
Foil Inertia/Ft Span	I_a	= 140.0 Slug Ft^2/ft
1/4 Chord Trailing Edge Flap		
Chord		= 2.145 ft
Wt.		= 1.75 tons
Flap Inertia/Ft Span	I_b	= 8.80 Slug Ft^2/ft
Keel to Foilborne Waterline Height	h	= 14.0 ft

IV. POWER

The first and most important component of overall control system power is the power to drive the hydrodynamic device through motions that will effect platforming in waves. A reasonable model of motion in the vertical plane for the hydrofoil ship is

$$M\ddot{z} + Mx_0\ddot{\theta} = Z(\text{waves}) + Z(\text{Ship Motion}) + Z(\text{Control Motion}) \qquad (1)$$

The effects of waves, ship motion, and control motion are modeled as linearly superimposed. For the platforming mode, the equation is reduced to

$$\text{Zero} = Z(\text{waves}) + Z(\text{control motion}) \qquad (2)$$

The objective is to cancel the lift caused by waves by generating an equal and opposite lift caused by the control surface. From the required motion of the control surface and the resultant forces on it, the power required can be calculated

$$\text{Power} = (\text{motion velocity})(\text{summation of forces in the direction of the velocity}) \qquad (3)$$

The next task is to look at the effect of waves on a foil.

A. QUASI-STEADY LIFT DUE TO WAVES

From a qualitative, quasi-steady point of view, the lift due to a wave passing a fully-submerged hydrofoil is

caused by the orbital velocity of the water particles superposed with the velocity of the foil through the water. For a steady foil, the orbital velocity appears as an angle of attack change for the incoming flow, Figure (9).

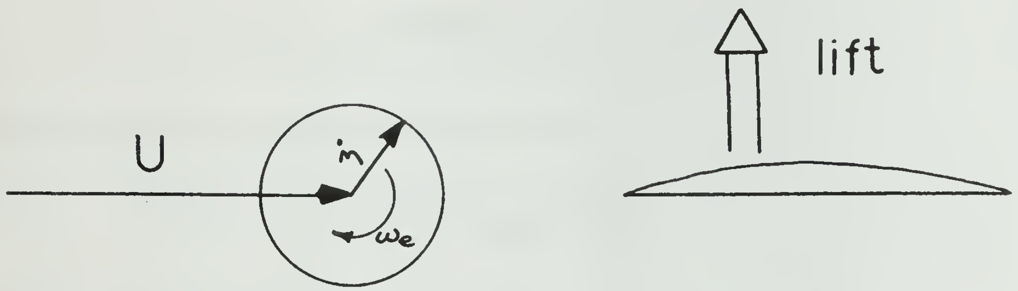


figure 9

For the hydrofoil ship, the limiting wave condition for continued platforming is--total amplitude of the wave equal to the keel-to-foilborne waterline height. For the particular hydrofoil ship chosen, this means a wave half-height of 7.0 feet. For an open-ocean wave, the common wave length for this amplitude is 280.0 feet. At the ocean surface, at a given point, a reasonable model of the wave vertical position is

$$\eta = \mathrm{Re}\{\eta_0 \exp(-i2\pi V/\lambda \cdot t)\}$$

For a deep water wave

$$V = \sqrt{g\lambda/2\pi}$$

or

$$\eta = \mathrm{Re}\{\eta_0 \exp(-i\sqrt{2\pi g/\lambda} \cdot t)\} \tag{6}$$

The wave vertical particle velocity is then

$$\dot{\eta} = \mathrm{Re}\{\eta_0(-i\sqrt{2\pi g/\lambda})\exp(-i\sqrt{2\pi g/\lambda} \cdot t)\} \tag{7}$$

The amplitude of the orbital velocity is then

$$|\dot{\eta}| = \eta_0\sqrt{2\pi g/\lambda} \tag{8}$$

For the chosen wave

$$|\dot{\eta}| = 5.85 \text{ ft/sec} \tag{9}$$

With the hydrofoil ship's velocity at 67.5 ft/sec, then the maximum angle of attack change due to <u>vertical</u> orbital velocity is

$$\mathrm{Tan}\ \alpha = 5.95/67.5 \tag{10}$$

or

$$\alpha \approx 5.06° \tag{11}$$

The maximum inflow velocity change due to <u>horizontal</u> orbital velocity is

$$U_1 = 67.5 \pm 5.95 \text{ ft/sec} \tag{12}$$

To determine the effect on lift, we must look at the lift coefficient C_l. For the 5.06° <u>angle of attack change</u>, flat plate model of the foil gives

$$\partial C_l/\partial\alpha \approx 2\pi \tag{13}$$

or

$$\Delta C_l \approx 2\pi\Delta\alpha \tag{14}$$

$$\Delta C_l \approx .546 \tag{15}$$

so

$$C_{l_1} \approx .4 + .546 = .946 \tag{16}$$

For the hydrofoil ship chosen, the ratio of the lifts is then

$$L_1/L_0 = 2.36 \tag{17}$$

In contrast, for the 5.95 ft/sec <u>inflow velocity change</u>, the ratio of the lifts is

$$L_1/L_0 = (U_1/U_0)^2 \tag{18}$$

$$L_1/L_0 \approx 1.19 \tag{19}$$

From the above, the effect on lift due to the vertical component of orbital velocity is far greater than that due to the horizontal component of orbital velocity. <u>This paper will only consider the effect of the vertical component of orbital velocity.</u>

The use of quasi-steady analysis is not adequate for the case of the hydrofoil ship chosen. The encounter frequency, semichord, and ship's velocity combine to give a reduced frequency

$$k = \omega_e b/U \tag{20}$$

in the range where unsteady effects are most important, as will be shown in Section (B). Figure (10) describes the concept of encounter frequency ω_e.

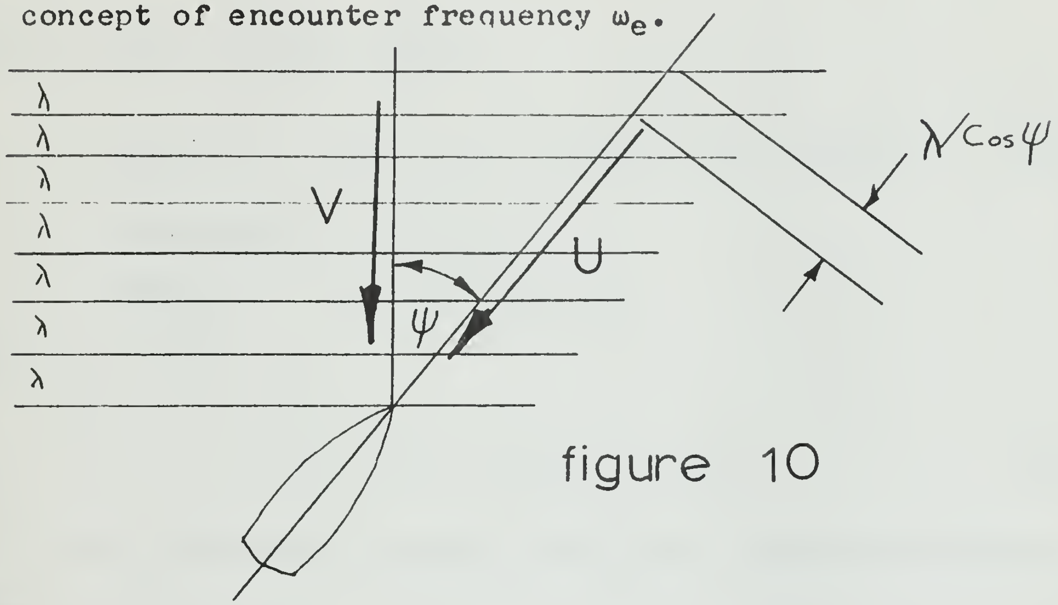


figure 10

here

$$\omega_e = 2\pi/T_e = 2\pi U_e/L_e \tag{21}$$

where

$$L_e = \lambda/\text{Cos}\psi \tag{22}$$

and

$$U_e = (V/\text{Cos}\psi) + U \tag{23}$$

so

$$\omega_e = (2\pi/\lambda)(V + U\text{Cos}\psi) \tag{24}$$

but

$$V^2 = g\lambda/2\pi \tag{5}$$

therefore

$$\lambda = 2\pi g/\omega_w^{\,2} \tag{26}$$

or

$$\omega_e = \omega_w + \omega_w^2 \cdot U\cos\psi/g \tag{27}$$

For the wave and ship chosen

$$\omega_e \approx .85 + .72(67.5)(1)/32.2 = 2.36 \text{ Rad/Sec} \tag{28}$$

Semichord $b = 4.29$ ft

Speed $U = 67.5$ ft/sec

so

$$k \approx 2.36(4.29)/67.5 = .15 \tag{29}$$

or k is in the most critical range for unsteady effects as shown in Section B.

B. UNSTEADY LIFT DUE TO WAVES

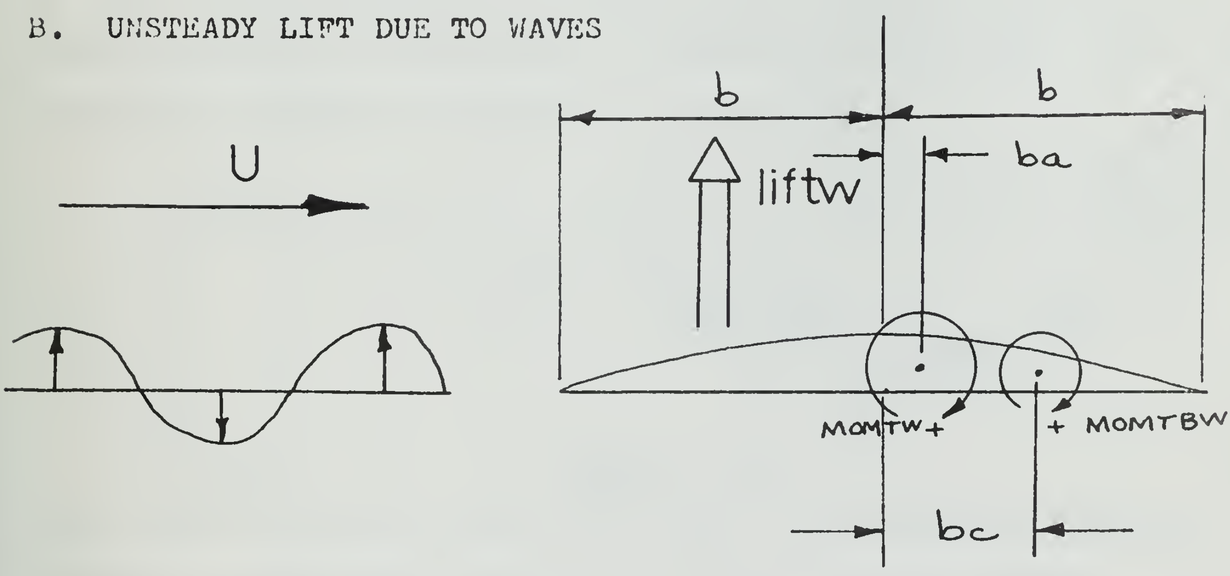


figure 11

The following is an adaptation of Chapter 5 of Reference (1) for this particular problem. Please refer to Table (3) in this paper for the following discussion. A model of a wave passing a steady foil, shown in Figure (11), is expressed by Equation (30). The disturbing upwash velocity $\overline{UPW}$ is equal to a complex phasor with magnitude--$|\dot{\eta}|$, phase angle (relative to wave height)---ϕ_W, and being driven around the phasor diagram with angular velocity--ω_e.

Assuming negligible effects from free surface, negligible energy radiation on the free surface; idealized 2-D flat plate theory, with no separation or cavitation, the unsteady lift is expressed as the quasi-steady lift modified by the Theodorsen Function and additional Bessel Functions. These functions are expressed in Equations (31 to 34). The Theodorsen Function is shown in Figure (12).

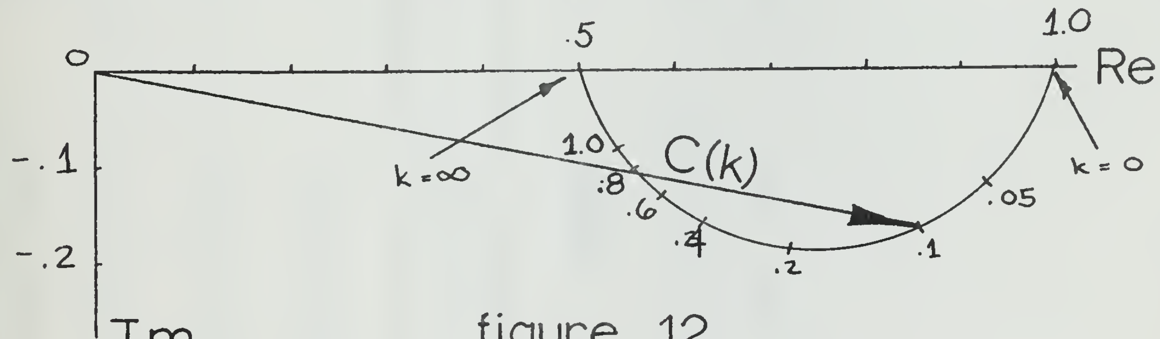


figure 12

From Figure (12), it can be seen that the Theodorsen Function reduces in magnitude (from 1 to 1/2) and has its maximum lagging phase angle change near k = .15;--exactly where the hydrofoil ship is operating.

TABLE (3)

$$\overline{UPW} = |\dot{\eta}|\exp(-i\omega_e t + \phi_W) \tag{30}$$

$$\overline{C(k)} = H_1^{(2)}(k)/[H_1^{(2)}(k) + iH_0^{(2)}(k)] \tag{31}$$

$$H_0^{(2)}(k) = J_0(k)-iY_0(k) \tag{32}$$

$$H_1^{(2)}(k) = J_1(k)-iY_1(k) \tag{33}$$

$$J_n(k) = \text{Bessel Function of Argument } k \tag{34}$$

$$\overline{LIFTW} = 2\pi\rho Ub\cdot\overline{UPW}\{\overline{C(k)}[J_0(k)-iJ_1(k)] + iJ_1(k)\} \tag{35}$$

$$\overline{MOMTW} = b(1/2 + a)\overline{LIFTW} \tag{36}$$

$$\overline{MOMTBW} = -b[(1-(1/2)c)\sqrt{1-c^2} + (1/2 - c)\mathrm{Cos}^{-1}c]\cdot\overline{LIFTW}/\pi \tag{37}$$

(Not found in Reference (1), see Appendix B.)

The expression for the complex phasor $\overline{LIFTW}$ is shown in Equation (35). The complex phasor moment $\overline{MOMTW}$ due to $\overline{LIFTW}$ acting about point ba (Figure (11)) is expressed in Equation (36). The complex phasor moment $\overline{MOMTBW}$ due to lift on the flap acting about point bc (Figure (11)) is expressed in Equation (37). These functions are best visualized with a phasor diagram. Figure (13) shows a phasor diagram scaled to illustrate the relationship between $\overline{UPW}$, $\overline{LIFTW}$, $\overline{MOMTW}$, and $\overline{MOMTBW}$ for $\omega_e = (+)$.

NOTE:

1. The diagram rotates <u>clockwise</u>.
2. The upwash velocity leads the wave height by 90°.
3. $\overline{LIFTW}$ lags $\overline{UPW}$.
4. $\overline{MOMTW}$ is in phase with $\overline{LIFTW}$.
5. $\overline{MOMTBW}$ is 180° out of phase with $\overline{LIFTW}$.
6. $\overline{LIFTW}$ and $\overline{MOMTW}$ would have been larger in magnitude and in phase with $\overline{UPW}$, if $k = 0$.

Figure (14) shows a phasor diagram again scaled to illustrate the same relationships for $\omega_e = (-)$.

NOTE:

1. The diagram rotates <u>counterclockwise</u>.
2. The upwash velocity lags the wave height by 90°.
3. Again $\overline{LIFTW}$ and $\overline{MOMTW}$ lag $\overline{UPW}$.
4. $\overline{MOMTBW}$ is 180° out of phase with $\overline{LIFTW}$.

Figure (15) illustrates the meaning of $\omega_e = (+,-)$.

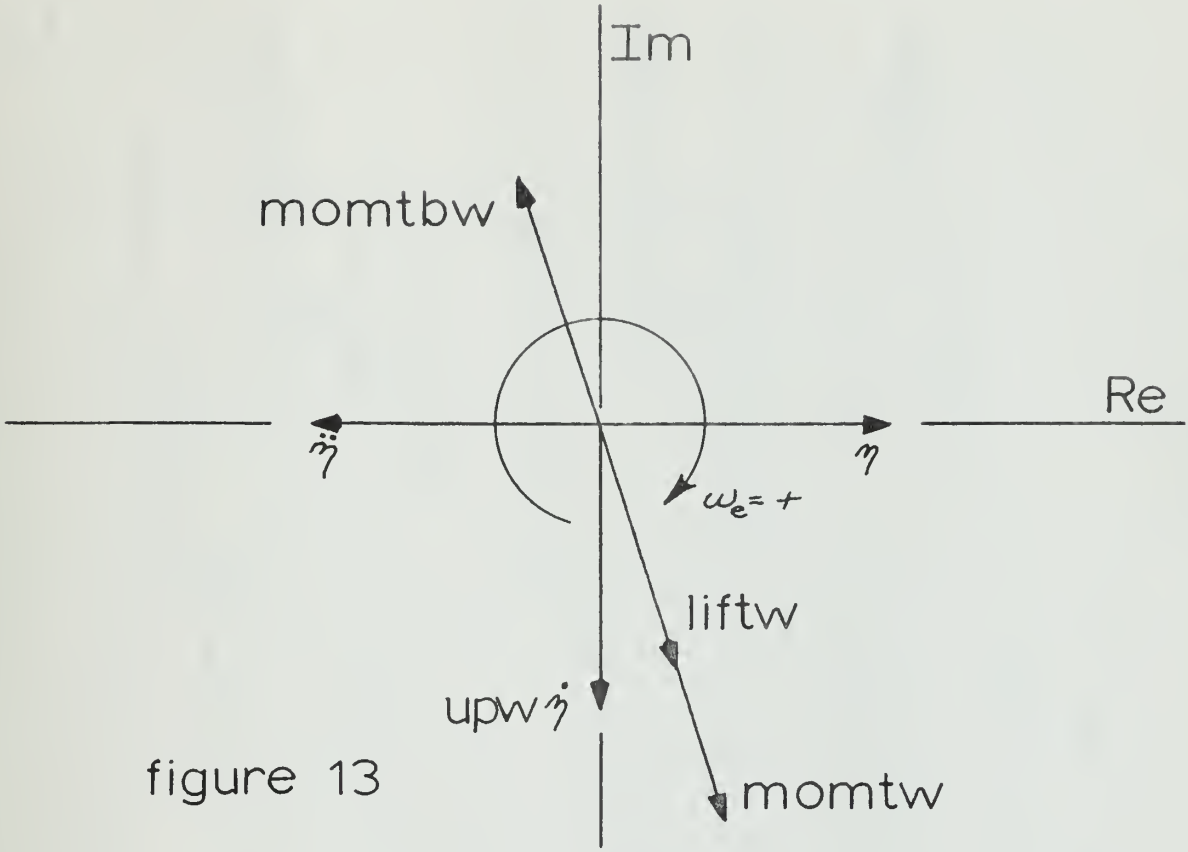


figure 13

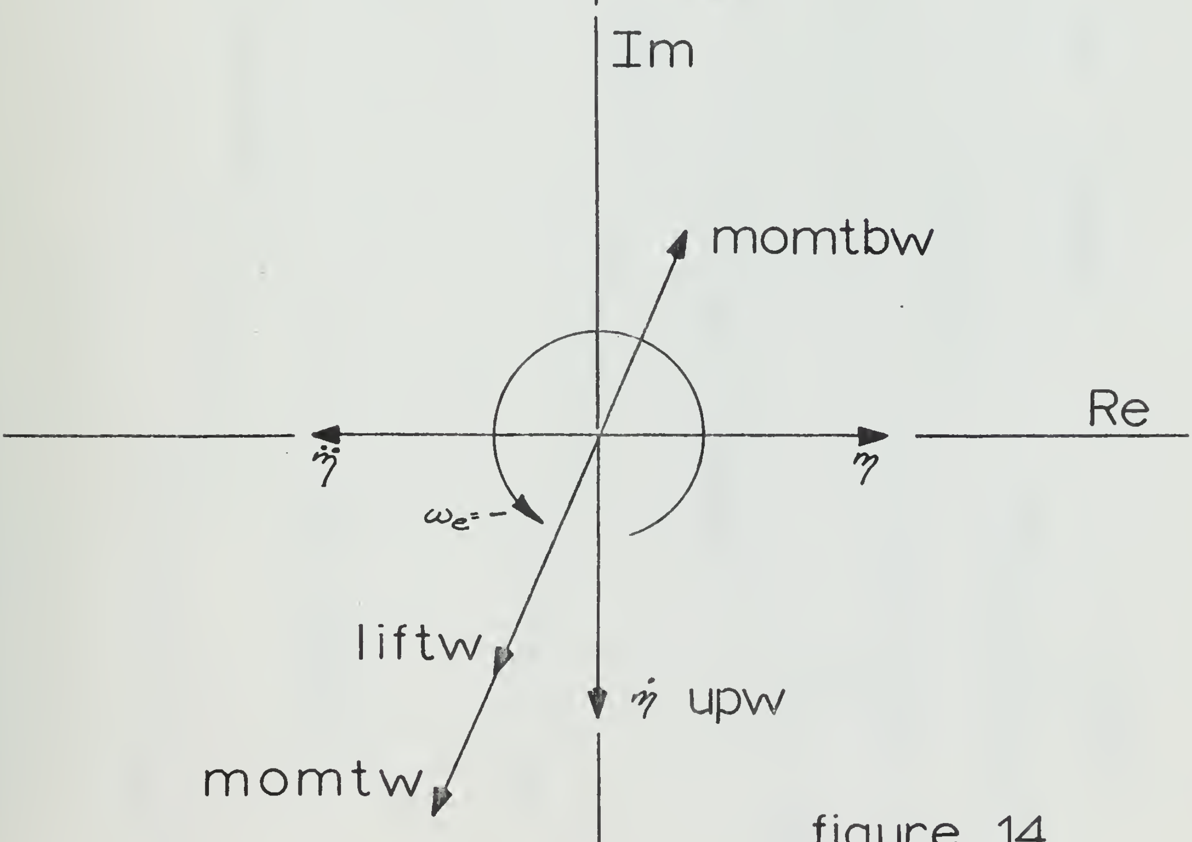
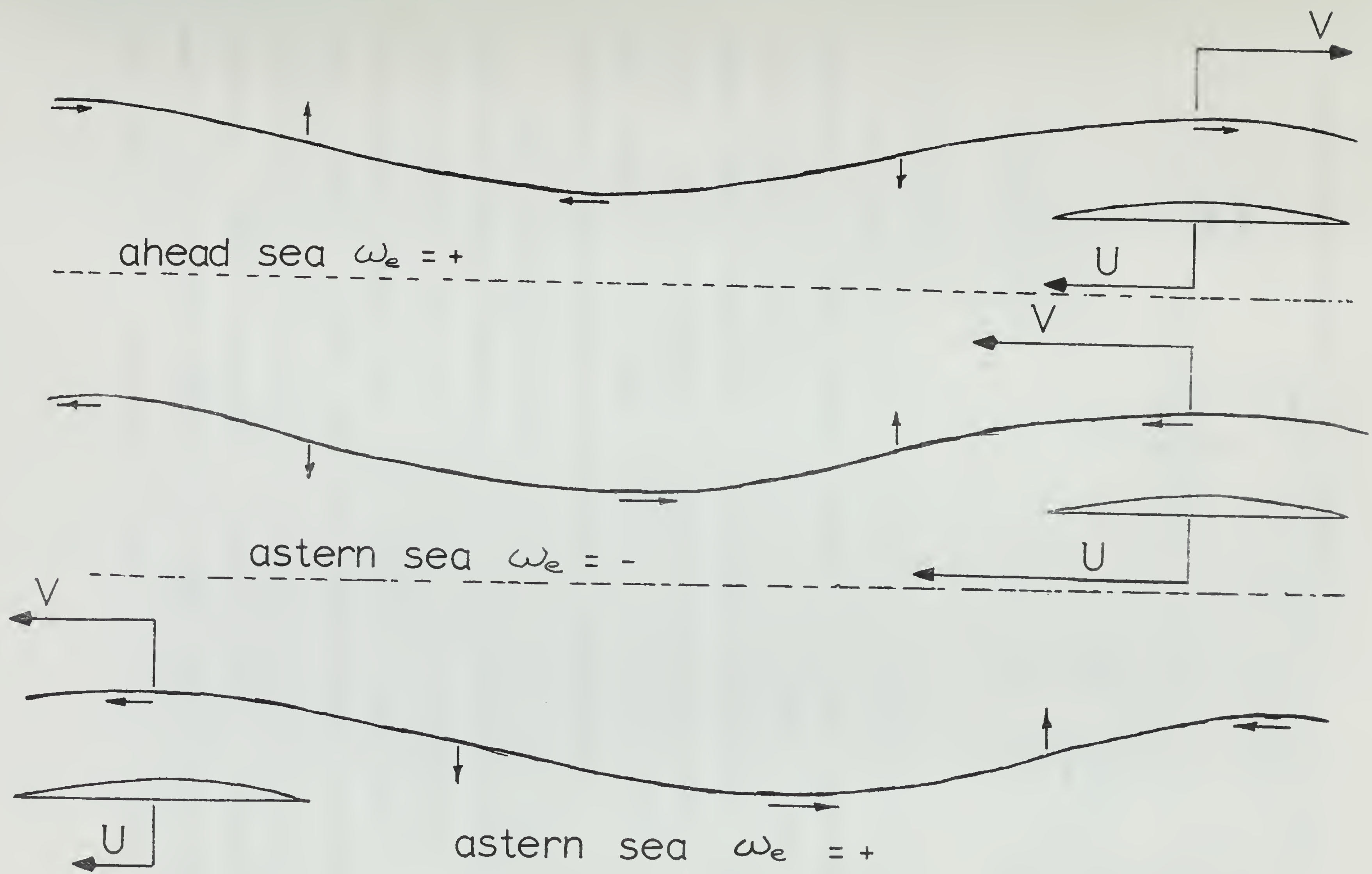


figure 14



V
ahead sea $\omega_e = +$
U
V
astern sea $\omega_e = -$
U
V
U
astern sea $\omega_e = +$

figure 15

C. UNSTEADY LIFT DUE TO AN OSCILLATING FOIL

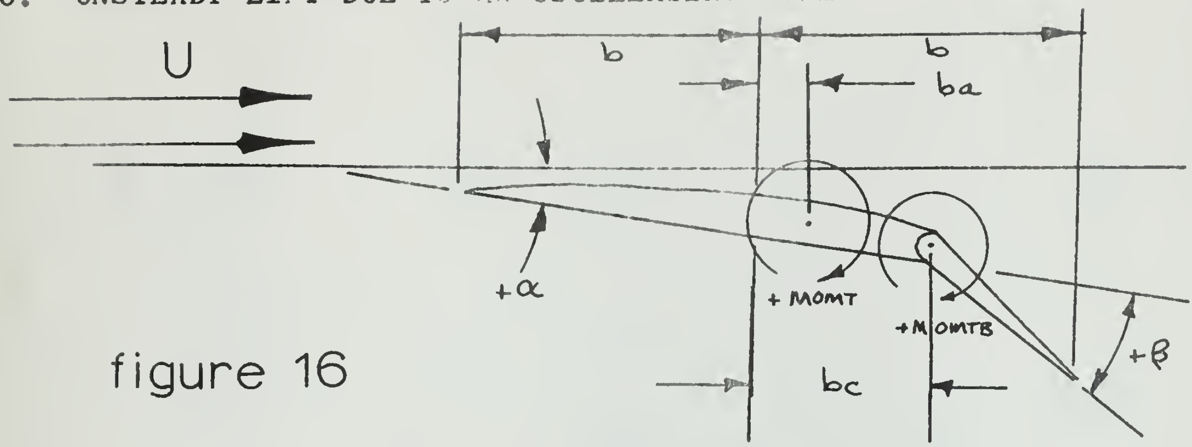


figure 16

1. Alpha Motion

The following is again an adaptation of Chapter 5 of Reference (1). Please refer to Table (4) in this paper for the following discussion. A model of full incidence motion (Alpha Motion) (Figure (16)) is expressed in Equations (38 to 40). It is expressed as a complex phasor with phase angle relative to the wave height--ϕ_1, and being driven around the phasor diagram with angular velocity--ω_e.

Again, assuming negligible effects from free surface, negligible energy radiation on the free surface; idealized 2-D flat plate theory, with no separation or cavitation, the unsteady complex phasor lift ($\overline{\text{LIFTA}}$) is expressed in non-circulatory velocity and acceleration terms, and circulatory position and velocity terms modified by the Theodorsen Function. $\overline{\text{LIFTA}}$ is expressed in Equation (41). The unsteady complex phasor moment $\overline{\text{MOMTA}}$ due to $\overline{\text{LIFTA}}$ acting about point ba (Figure (16)) is expressed in Equation (42). The unsteady

TABLE (4)

$$\overline{ALPHA} = \alpha_0 \exp(-i\omega_e t + \phi_1) \tag{38}$$

$$\overline{ALPHADOT} = \alpha_0(-i\omega_e)\exp(-i\omega_e t + \phi_1) \tag{39}$$

$$\overline{ALPHADDOT} = \alpha_0(-\omega_e{}^2)\exp(-i\omega_e t + \phi_1) \tag{40}$$

$$\overline{LIFTA} = \pi\rho b^2[U\cdot\overline{ALPHADOT} - ba\cdot\overline{ALPHADDOT}] \Big|_{NC}$$

$$+2\pi\rho Ub\overline{C(k)}[U\cdot\overline{ALPHA} + b(1/2 - a)\cdot\overline{ALPHADOT}] \Big|_{C} \tag{41}$$

$$\overline{MOMTA} = \pi\rho b^2[U^2\cdot\overline{ALPHA} - b^2(1/8 + a^2)\cdot\overline{ALPHADDOT}] \Big|_{NC}$$

$$-2\pi\rho Ub^2[1/2 - (a+1/2)\overline{C(k)}]\{U\cdot\overline{ALPHA} + b(1/2 - a)\cdot\overline{ALPHADOT}\} \Big|_{C} \tag{42}$$

$$\begin{aligned}\overline{MOMTBA} = & -\rho b^2U^2\cdot\overline{ALPHA}[c\sqrt{1-c^2} - \mathrm{Cos}^{-1}c] \\ & +\rho b^3U\cdot\overline{ALPHADOT}\{a[c\sqrt{1-c^2} - \mathrm{Cos}^{-1}c] + (1/3)(\sqrt{1-c^2})^3 \\ & - (1/3)(2+c^2)\sqrt{1-c^2} + c\mathrm{Cos}^{-1}c\} \\ & -\rho b^4\cdot\overline{ALPHADDOT}\{(1/8 + c^2)\mathrm{Cos}^{-1}c - (1/8c\sqrt{1-c^2}(7+2c^2) \\ & + (c-a)[(1/3)(2+c^2)\sqrt{1-c^2} - c\mathrm{Cos}^{-1}c]\} \Bigg|_{NC}\end{aligned}$$

$$\begin{aligned}& -2\rho Ub^2\{(1/2)[\mathrm{Cos}^{-1}c - c\sqrt{1-c^2}] + [(1+c/2)\sqrt{1-c^2} \\ & - (c+1/2)\mathrm{Cos}^{-1}c]\overline{C(k)}\}\cdot\{U\cdot\overline{ALPHA} + b(1/2 - a)\cdot\overline{ALPHADOT}\} \Bigg|_{C}\end{aligned} \tag{43}$$

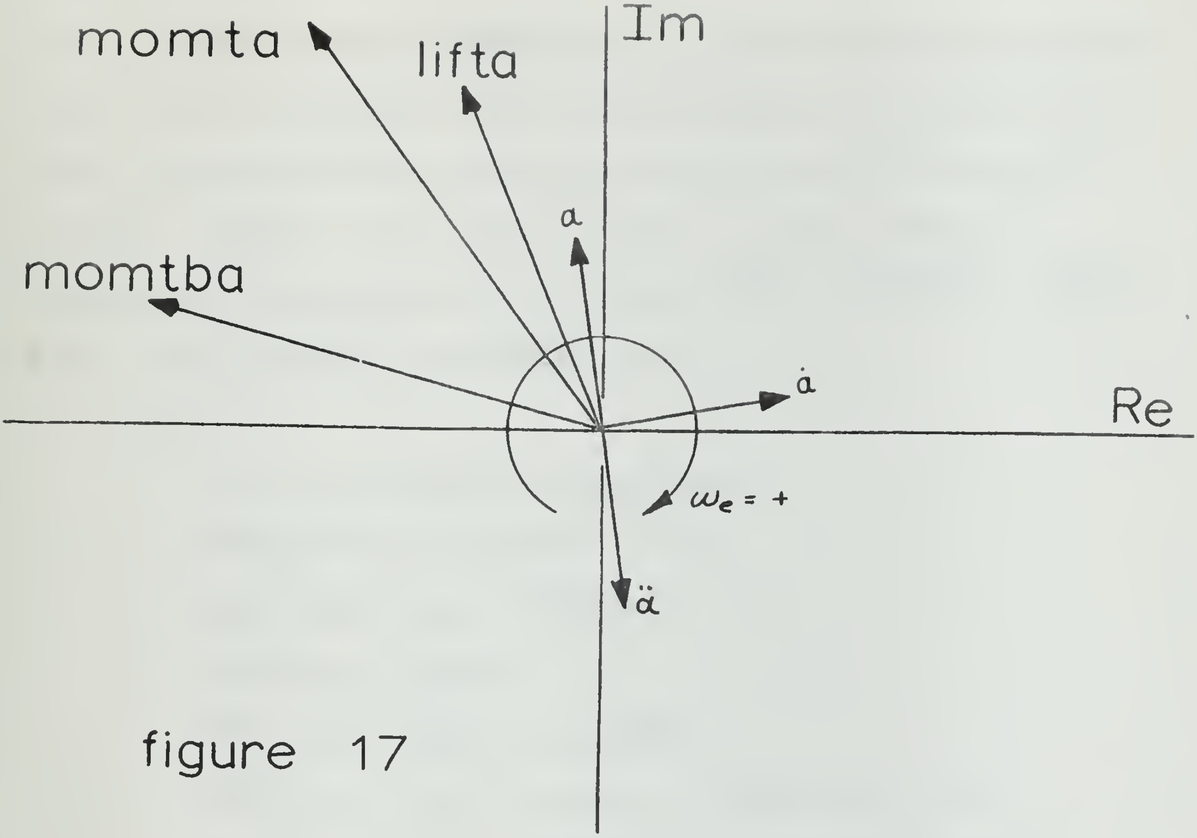


figure 17

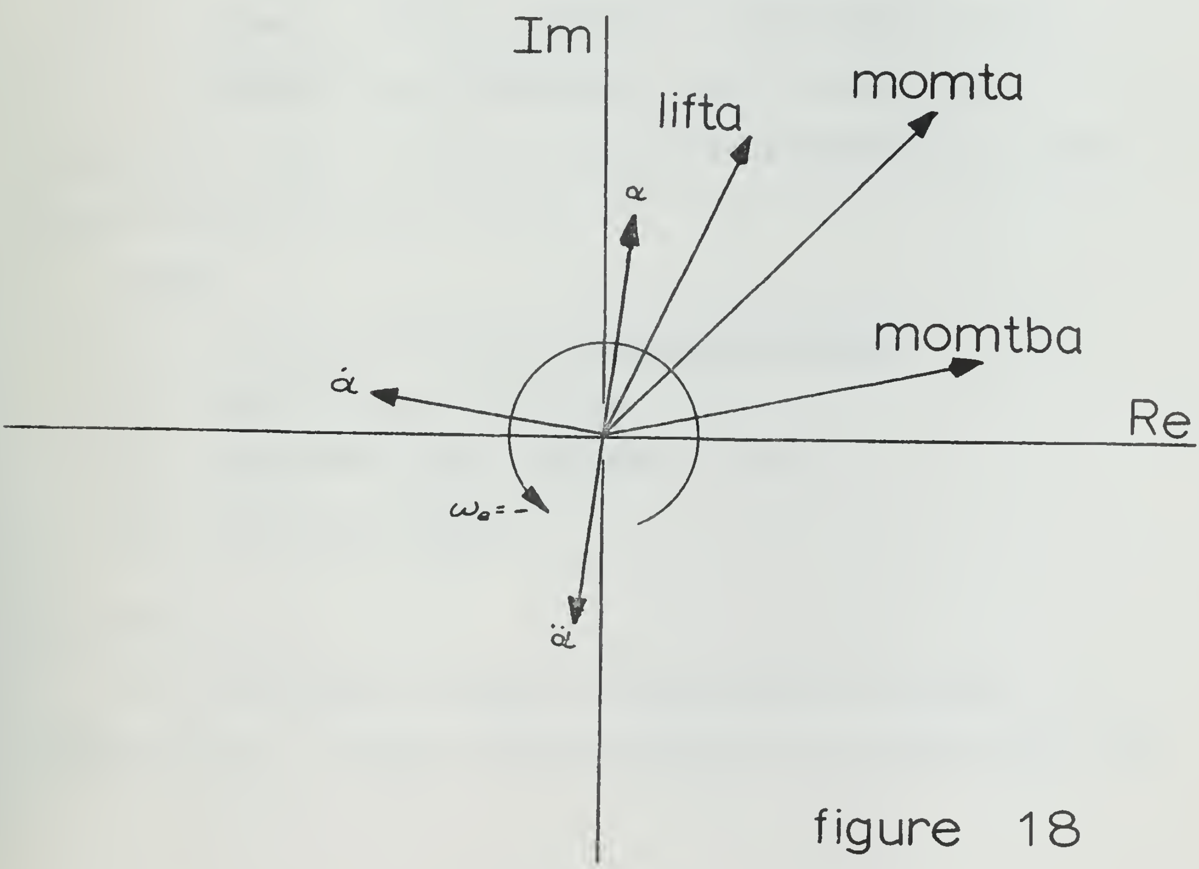


figure 18

complex phasor moment $\overline{\text{MOMTBA}}$ due to lift on the flap acting about point bc (Figure (16)) is expressed in Equation (43). Again, these functions are best visualized with a phasor diagram. Figure (17) shows a phasor diagram scaled to illustrate the relationship between $\overline{\text{ALPHA}}$, $\overline{\text{ALPHADOT}}$, $\overline{\text{ALPHADDOT}}$, $\overline{\text{LIFTA}}$, $\overline{\text{MOMTA}}$, and $\overline{\text{MOMTBA}}$ for $\omega_e = (+)$.

NOTE:

1. The diagram rotates clockwise.
2. $\overline{\text{ALPHADOT}}$ leads $\overline{\text{ALPHA}}$ by 90°.
3. $\overline{\text{ALPHADDOT}}$ leads $\overline{\text{ALPHADOT}}$ by 90°.
4. $\overline{\text{LIFTA}}$ lags $\overline{\text{ALPHA}}$.
5. $\overline{\text{MOMTA}}$ and $\overline{\text{MOMTBA}}$ lag $\overline{\text{LIFTA}}$.
6. $\overline{\text{LIFTA}}$ will be in phase with $\overline{\text{ALPHA}}$ for k = 0 because although $\overline{\text{LIFTA}}$ has contributions from $\overline{\text{ALPHADOT}}$ and $\overline{\text{ALPHADDOT}}$, these are zero for k = 0.

Figure (18) shows a phasor diagram to illustrate the same relationships with $\omega_e = (-)$.

NOTE:

1. The diagram rotates counterclockwise.
2. $\overline{\text{ALPHADOT}}$ leads $\overline{\text{ALPHA}}$ by 90°.
3. $\overline{\text{ALPHADDOT}}$ leads $\overline{\text{ALPHADOT}}$ by 90°.
4. $\overline{\text{LIFTA}}$ lags $\overline{\text{ALPHA}}$.

2. Beta Motion

The following is again an adaptation of Chapter 5 of Reference (1). Please refer to Table (5) in this paper for

TABLE (5)

$$\overline{BETA} = (\beta_0/\alpha_0)\alpha_0 \exp(-i\omega_e t + \phi_1 + \Delta\phi) \tag{44}$$

$$\overline{BETADOT} = (\beta_0/\alpha_0)\alpha_0(-i\omega_e)\exp(-i\omega_e t + \phi_1 + \Delta\phi) \tag{45}$$

$$\overline{BETADDOT} = (\beta_0/\alpha_0)\alpha_0(-\omega_e{}^2)\exp(-i\omega_e t + \phi_1 + \Delta\phi) \tag{46}$$

$$\begin{aligned}
\overline{LIFTB} = {} & \left. \begin{aligned} & -\rho b^2 U\cdot\overline{BETADOT}[c\sqrt{1-c^2} - \mathrm{Cos}^{-1}c] \\ & -\rho b^3\cdot\overline{BETADDOT}[c\,\mathrm{Cos}^{-1}c - (1/3)(2+c^2)\sqrt{1-c^2}] \end{aligned} \right|_{NC} \\
& \left. \begin{aligned} & +2\pi\rho Ub\overline{C(k)}U\cdot\overline{BETA}[\sqrt{1-c^2} + \mathrm{Cos}^{-1}c]/\pi \\ & +2\pi\rho Ub\overline{C(k)}b\cdot\overline{BETADOT}[(1-2c)\mathrm{Cos}^{-1}c + (2-c)\sqrt{1-c^2}]/2\pi \end{aligned} \right|_{C}
\end{aligned} \tag{47}$$

$$\begin{aligned}
\overline{MOMTB} = {} & \left. \begin{aligned} & -\rho b^2 U^2\cdot\overline{BETA}[c\sqrt{1-c^2} - \mathrm{Cos}^{-1}c] \\ & -\rho b^3 U\cdot\overline{BETADOT}\{(1/3)\sqrt{1-c^2}(c^2-1) - (c-a)[c\sqrt{1-c^2} - \mathrm{Cos}^{-1}c]\} \\ & -\rho b^4\cdot\overline{BETADDOT}\{(1/8 + c^2)\mathrm{Cos}^{-1}c - (1/8)c\sqrt{1-c^2}(7+2c^2) \\ & + (c-a)[(1/3)\sqrt{1-c^2}(2+c^2) - c\,\mathrm{Cos}^{-1}c]\} \end{aligned} \right|_{NC} \\
& \left. \begin{aligned} & -2\pi\rho Ub^2[1/2 - (a+1/2)\overline{C(k)}]\{U\cdot\overline{BETA}(\sqrt{1-c^2} + \mathrm{Cos}^{-1}c)/\pi \\ & + b\cdot\overline{BETADOT}[(1-2c)\mathrm{Cos}^{-1}c + (2-c)\sqrt{1-c^2}]/2\pi\} \end{aligned} \right|_{C}
\end{aligned} \tag{48}$$

TABLE (5) (CONT'D)

$$
\begin{aligned}
\overline{\text{MOMTBB}} = {} & \left. \begin{aligned} & -\rho b^2 U^2 \cdot \overline{\text{BETA}}[2c\sqrt{1-c^2}\,\text{Cos}^{-1}c - (1-c^2) - (\text{Cos}^{-1}c)^2]/\pi \\ & +\rho b^4 \cdot \overline{\text{BETADDOT}}[(1/4)c\sqrt{1-c^2}\,\text{Cos}^{-1}c(7+2c^2) \\ & - (1/8 + c^2)(\text{Cos}^{-1}c)^2 - (1/8)(1-c^2)(5c^2+4)]/\pi \end{aligned} \right|_{NC} \\
& \left. \begin{aligned} & -2\rho U b^2\{(1/2)[\text{Cos}^{-1}c - c\sqrt{1-c^2}] + [(1+c/2)\sqrt{1-c^2} \\ & - (c+1/2)\text{Cos}^{-1}c]\overline{C(k)}\}\cdot\{U\cdot\overline{\text{BETA}}[\sqrt{1-c^2} + \text{Cos}^{-1}c]/\pi \\ & +b\cdot\overline{\text{BETADOT}}[(1-2c)\text{Cos}^{-1}c + (2-c)\sqrt{1-c^2}]/2\pi\} \end{aligned} \right|_{C}
\end{aligned}
\qquad (49)
$$

the following discussion. A model of trailing edge flap motion (Beta Motion) (Figure (16)) is expressed in Equations (44 to 46). It is expressed in terms of the complex phasor $\overline{\text{ALPHA}}$. The ratio of magnitude between $\overline{\text{BETA}}$ and $\overline{\text{ALPHA}}$ is--β_0/α_0; the phase difference between $\overline{\text{BETA}}$ and $\overline{\text{ALPHA}}$ (+ for $\overline{\text{BETA}}$ leading) is--$\Delta\phi$; and the phasor is being driven around the phasor diagram with angular velocity--ω_e.

Again, assuming negligible effects from free surface, negligible energy radiation on the free surface, idealized 2-D flat plate theory; with no separation or cavitation, the unsteady complex phasor lift ($\overline{\text{LIFTB}}$) is expressed in non-circulatory velocity and acceleration terms, and circulatory position and velocity terms modified by the Theodorsen Function. $\overline{\text{LIFTB}}$ is expressed in Equation (47). The unsteady complex phasor moment $\overline{\text{MOMTB}}$ due to $\overline{\text{LIFTB}}$ acting about point ba (Figure (16)) is expressed in Equation (47). The unsteady complex phasor moment $\overline{\text{MOMTBB}}$ due to lift on the flap acting about point bc (Figure (16)) is expressed in Equation (49). Again, these functions are best visualized with a phasor diagram. Figure (19) shows a phasor diagram scaled to illustrate the relationship between $\overline{\text{BETA}}$, $\overline{\text{BETADOT}}$, $\overline{\text{BETADDOT}}$, $\overline{\text{LIFTB}}$, $\overline{\text{MOMTB}}$, and $\overline{\text{MOMTBB}}$ for ω_e = (+).

NOTE: Same as Figure (17).

Figure (20) shows a phasor diagram to illustrate the same relationships with ω_e = (-).

NOTE: Same as Figure (18).

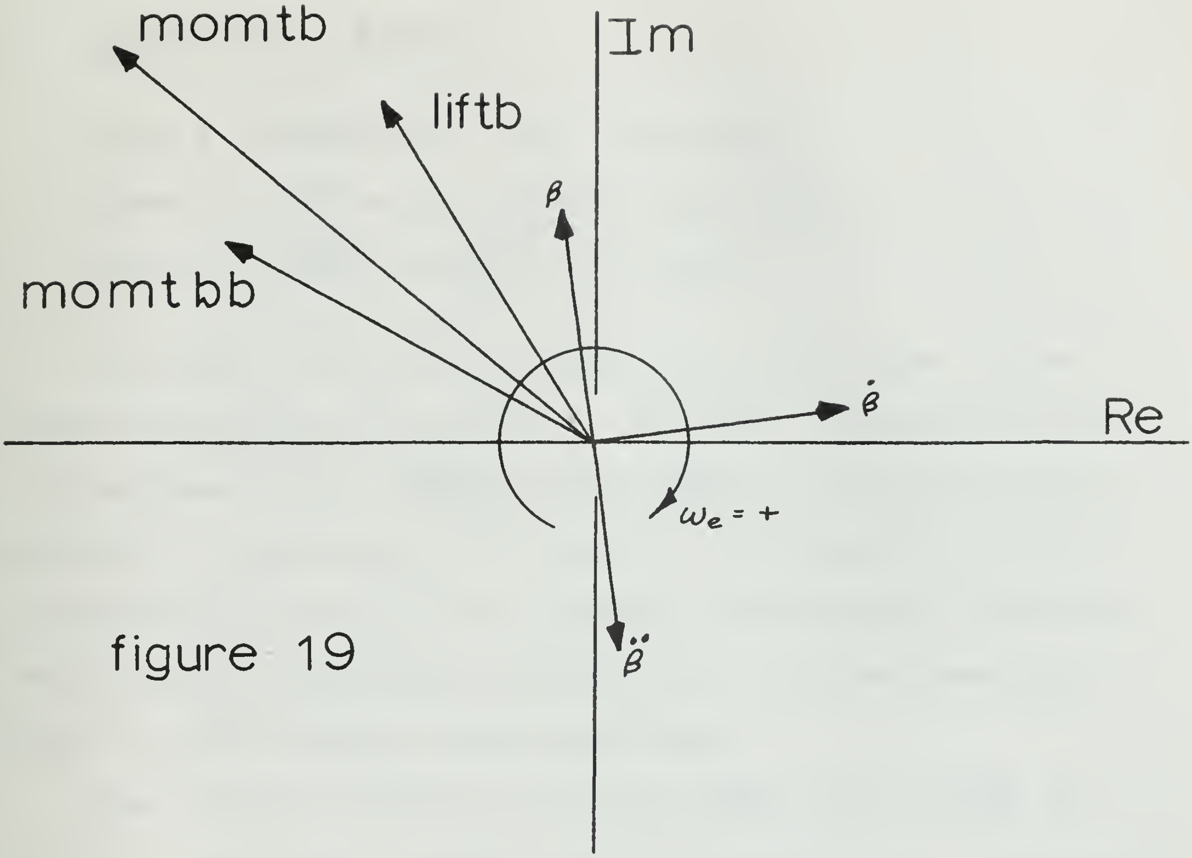


figure 19

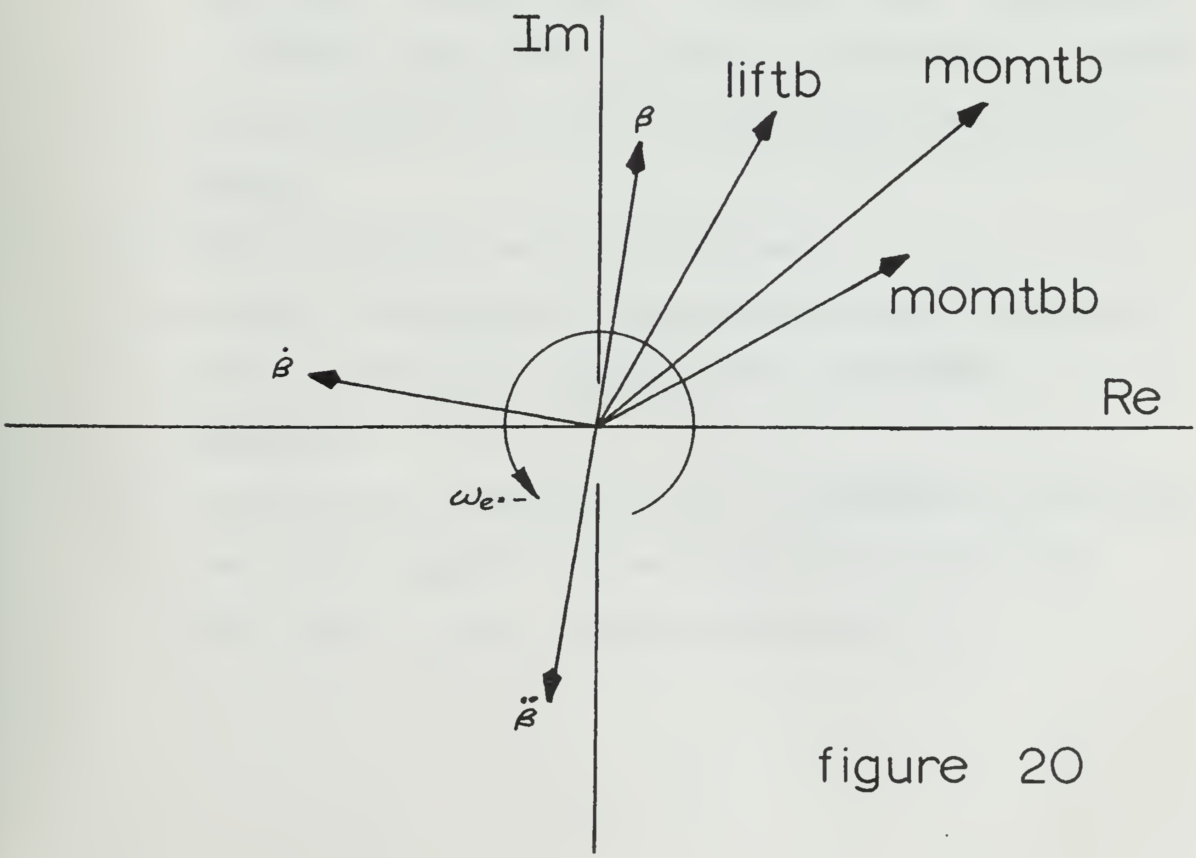


figure 20

D. REQUIRED FOIL MOTION

Mode 1 = $\overline{\text{ALPHA}}$ only (Full Incidence)

Mode 2 = $\overline{\text{BETA}}$ only (Trailing Edge Flap)

Mode 3 = $\overline{\text{ALPHA}}$ and $\overline{\text{BETA}}$ (Tab Foil)

The wave is given. The lift due to the wave can be solved explicitly by Section B. However, the required control surface motion ($\overline{\text{ALPHA}}$ and/or $\overline{\text{BETA}}$) is only implicitly expressed in Equations (41) and (47) of Section C. Both Equations (41) and (47) are complex expressions. Each one represents two equations either as the real and imaginary parts or the magnitude and phase angle.

The strategy used to solve for $\overline{\text{ALPHA}}$ and/or $\overline{\text{BETA}}$ is:

1. Substitute in Equations (41) and/or (47) expressions for $\overline{\text{ALPHA}}$ and/or $\overline{\text{BETA}}$ and their derivatives in terms of an unknown α_0 and ϕ_1. (β_0/α_0 and $\Delta\phi$ known constants)
2. Expand multiple angle expressions.
3. Separate the real and imaginary parts of the modified Equations (41) and/or (47). (-Re$\overline{\text{LIFTW}}$, -Im$\overline{\text{LIFTW}}$)
4. Divide out α_0, $\text{Cos}\phi_1$, ρ, and b. (-Re$\overline{\text{LIFTW}}/(\alpha_0 \text{Cos}\phi_1 \rho b)$.
5. Group the remaining terms into terms containing Tan ϕ_1--Y_1, X_1, and constant terms--Y_2, X_2.

$$-Re\overline{LIFTW}/(\alpha_0 Cos\phi_1 \rho b) = Tan\phi_1(Y_1) + (Y_2) \tag{50}$$

$$-Im\overline{LIFTW}/(\alpha_0 Cos\phi_1 \rho b) = Tan\phi_1(X_1) + (X_2) \tag{51}$$

6. Divide the real equation by the imaginary equation.

$$\begin{aligned} -Re\overline{LIFTW}/(-Im\overline{LIFTW}) &= R \\ &= (Tan\phi_1(Y_1)+(Y_2))/(Tan\phi_1(X_1) + (X_2)) \end{aligned} \tag{52}$$

7. Separate out Tan ϕ_1

$$Tan\ \phi_1 = (Y_2-RX_2)/(RX_1-Y_1) \tag{53}$$

8. Solve for ϕ_1.

9. Magnitude α_0 is

$$\alpha_0 = -Re\overline{LIFTW}/[Cos\phi_1 \rho b(Tan\phi_1(Y_1) + (Y_2))] \tag{54}$$

10. The problem is solved.

See Appendix C for details. With $\overline{ALPHA}$ and $\overline{BETA}$, the equations in Section C will give lift and moment from the motion.

E. SUMMATION OF HYDRODYNAMIC FORCES

Summation of $\overline{LIFTW}$, $\overline{LIFTA}$ and/or $\overline{LIFTB}$ should equal zero. Summation of $\overline{MOMTW}$, $\overline{MOMTA}$ and/or $\overline{MOMTB}$ gives the total hydrodynamic moment about ba. Summation of $\overline{MOMTBW}$, $\overline{MOMTBA}$ and/or $\overline{MOMTBB}$ gives the total hydrodynamic moment acting on the flap about bc.

F. REQUIRED APPLIED MOMENTS

The control system must apply a moment ($\overline{MOMAP}$) on the control surface to produce the required motion. The dynamics of this interaction is expressed in Equations (55) and (56).

$$I_a \cdot \overline{ALPHADDOT} = \overline{MOMTW} + \overline{MOMTA} + \overline{MOMTB} + \overline{MOMAPA} \quad (55)$$

$$I_b \cdot \overline{BETADDOT} = \overline{MOMTBW} + \overline{MOMTBA} + \overline{MOMTBB} + \overline{MOMAPB} \quad (56)$$

From these equations, the required applied moments can be found. Figure (21) shows these relationships in phasor form.

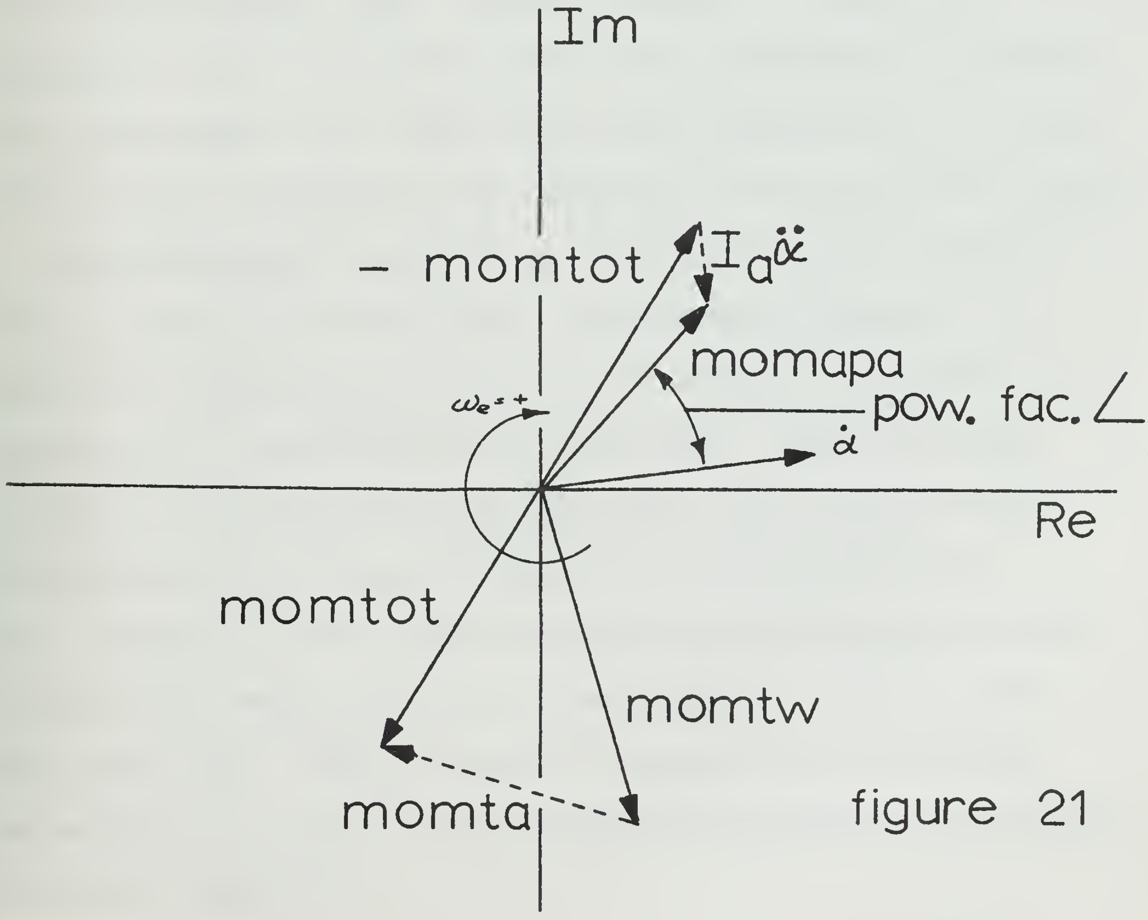


figure 21

G. AVERAGE POWER

The average power (PAVE) required to produce the motion is expressed in Equations (57) and (58).

$$PAVEA = |\overline{ALPHADOT}| \cdot |\overline{MOMAPA}| \cdot \text{POWER FACTOR}/2 \tag{57}$$

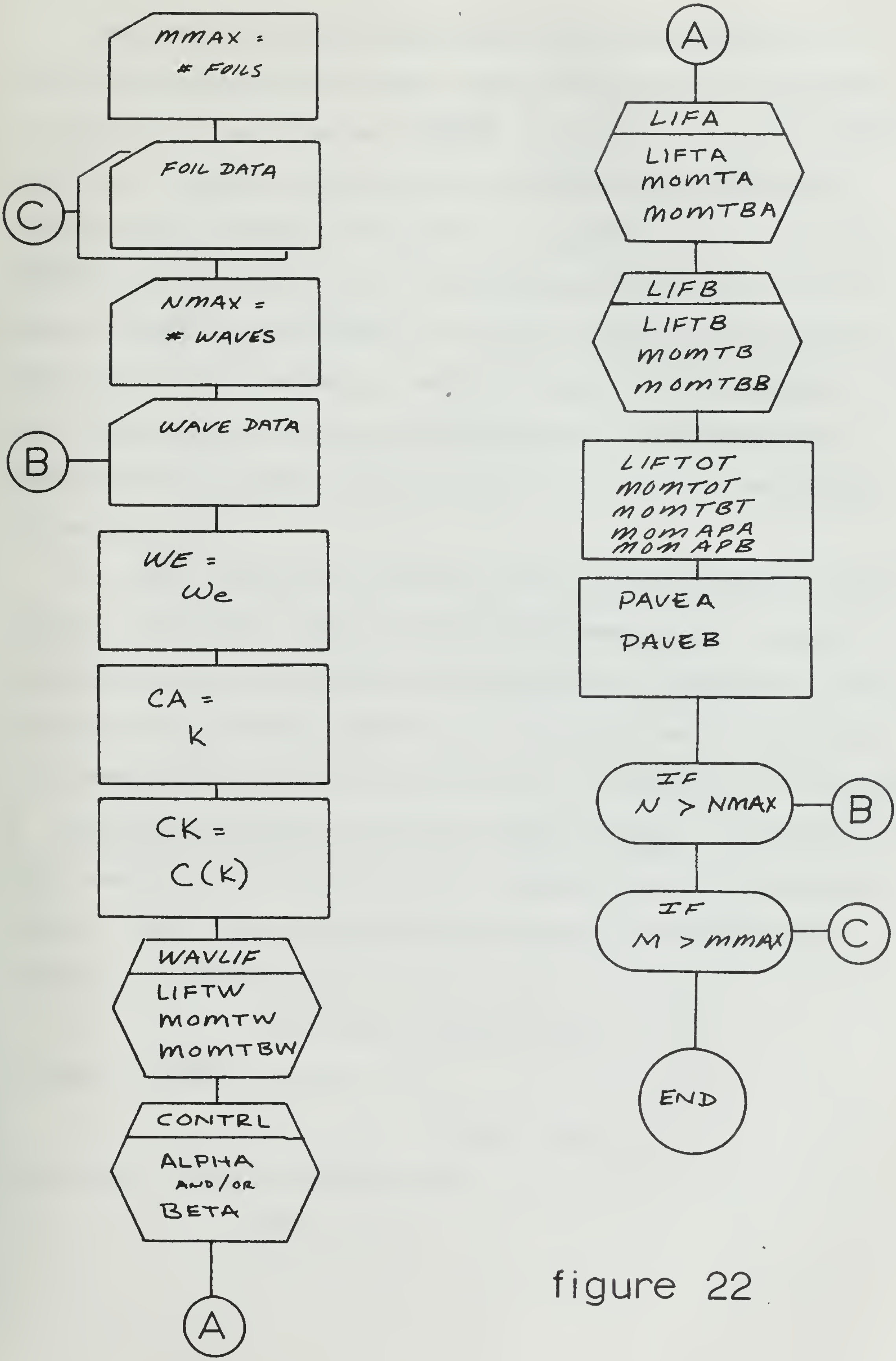
$$PAVEB = |\overline{BETADOT}| \cdot |\overline{MOMAPB}| \cdot \text{POWER FACTOR}/2 \tag{58}$$

where the power factor is the cosine of the phase angle between the motion velocity and the applied moment.

H. PROGRAM

Two programs were written in Fortran IV for use at the Information Processing Center on the "IBM System 360". The first program writes out the details of the power calculation, and the second writes out a minimum of identifying information and plots the power results on a "Calcomp Plotter". An outline of the program is shown on Figure (22).

The encounter frequency is calculated using Equation (27) as shown in Figure (10). The reduced frequency is calculated using Equation (20) but taking the absolute magnitude for use with Bessel Functions. The Theodorsen Function is calculated using Equation (31) by calling the "IBM Scientific Subroutine Package" routines BESJ and BESY. Because of the iterative method of calculation used in BESY, the reduced frequency parameter k can not be less than 0.05. This is guarded against in the program but results in errors in the Theodorsen Function for reduced frequency below 0.05.



mmAX =
* FOILS
FOIL DATA
C
NmAX =
* WAVES
WAVE DATA
B
WE =
We
CA =
K
CK =
C(K)
WAVLIF
LIFTW
MOMTW
MOMTBW
CONTRL
ALPHA
AND/OR
BETA
A
A
LIFA
LIFTA
MOMTA
MOMTBA
LIFB
LIFTB
MOMTB
MOMTBB
LIFTOT
MOMTOT
MOMTBT
MOMAPA
MOMAPB
PAVEA
PAVEB
IF
N > NMAX
B
IF
M > MMAX
C
END

figure 22

The WAVLIF routine calculates the complex phasor forces generated by the wave using the analysis of Section B. The lift due to waves ($\overline{\text{LIFTW}}$) is brought into the CONTRL routine. Here the complex phasor $\overline{\text{ALPHA}}$ and/or $\overline{\text{BETA}}$ are calculated to produce an equal and opposite lift to cancel $\overline{\text{LIFTW}}$. The procedure used is described in Section D. The motion ($\overline{\text{ALPHA}}$ and/or $\overline{\text{BETA}}$) is brought into the two routines LIFA and LIFB to calculate the lift caused by the motion, 1) as a check on the results of CONTRL; and 2) in order to calculate the moments about ba and bc, using the equations of Section C.

So far, the control surface motion and the hydrodynamic moments are known. The hydrodynamic moments due to waves and control surface motion are summed as in Section E. Next, the required applied moments are calculated as in Section F. The average power is calculated using the analysis of Section G. <u>Note all results are 2-D per foot of span.</u> An important requirement is the compatibility between mode specification and the input data values for BA = β_0/α_0, and DPHE = $\Delta\phi$.

For Mode 1 $\overline{\text{ALPHA}}$ motion,

BA = 0.0   DPHE = 0.0

For Mode 2 $\overline{\text{BETA}}$ motion,

BA = 1.0   DPHE = 0.0

For Mode 3 $\overline{\text{ALPHA}}$ and $\overline{\text{BETA}}$ motion

DPHE $\neq$ 0.0

The plotting routines plot the motion variable magnitude and phase angle, and the average power as functions of encounter frequency. One plot per foil, the plotter treats each wave run as a point and connects each point with a straight line. The choice of wave input data should be arranged to give a somewhat continuous variation as was done for the series discussed in Section I.

Appendix D contains a listing of the programs and an example of their written output. Examples of the plotted output can be found in Section I.

I. RESULTS

The two pieces of input data required for each run of the program are the foil data and the wave data. Three sets of calculations were performed for the three candidate foils, as described by Table (2):

Mode 1 (Full Incidence)

Mode 2 (Trailing Edge Flap)

Mode 3 (Tab Foil)

The first set is a series of waves with ship speed constant, wave length constant, and angle of attack (SI) between the ship and the waves varying from "dead ahead" to "dead astern". Table (6) displays the wave parameters for each run of this set. Figures (23), (24), and (25) are the computer plotted results for the three foils run with this series. The series can be pictured as a slow 180° turn being executed by the ship at constant speed in regular waves of constant wave length.

TABLE (6)

SHIP ANGLE OF ATTACK TO THE WAVE, VARIATION

RUN #	HW=η (FT)	WL=λ (FT)	U=U (FT/SEC)	SI=ψ (RADS)	PHEW=ϕw (RADS)
1	7.0	280.0	80.0	0.0	-1.5708
2	7.0	280.0	80.0	.3927	-1.5708
3	7.0	280.0	80.0	.7854	-1.5708
4	7.0	280.0	80.0	1.1781	-1.5708
5	7.0	280.0	80.0	1.3745	-1.5708
6	7.0	280.0	80.0	1.5708	-1.5708
7	7.0	280.0	80.0	1.7672	-1.5708
8	7.0	280.0	80.0	1.9635	-1.5708
9	7.0	280.0	80.0	2.1599	-1.5708
10	7.0	280.0	80.0	2.3562	-1.5708
11	7.0	280.0	80.0	2.5526	-1.5708
12	7.0	280.0	80.0	2.7489	-1.5708
13	7.0	280.0	80.0	2.9453	-1.5708
14	7.0	280.0	80.0	3.1416	-1.5708

The second set is a series of <u>waves</u> with ship speed (U) varying from 90 ft/sec down to 25 ft/sec, wave length constant, and angle of attack between the ship and the waves at "dead ahead" and "dead astern". Table (7) displays the <u>wave</u> parameters for each run of this set. Figures (26), (27), and (28) are the computer plotted results for the three <u>foils</u> run with this series. The series can be pictured as a slow deceleration by the ship in ahead regular waves of constant wave length and a slow acceleration by the ship in astern regular waves of constant wave length.

The third set is a series of <u>waves</u> with constant ship speed, wave length (WL) varying from 280 ft to 140 ft, and angle of attack between the ship and the waves at "dead ahead" and "dead astern". Table (8) displays the <u>wave</u> parameters for each run of this set. Figures (29), (30), and (31) are the computer plotted results for the three <u>foils</u> run with this series. The series can be pictured as a slow transit by the ship at constant speed in ahead regular waves of reducing wave length and a slow transit in astern regular waves of increasing wave length.

Additional trial and error runs were made in order to discover a useful motion ratio β_0/α_0 (BA) and a useful phase angle difference $\Delta\phi$ (DPHE) for Mode 3 (Tab Foil). The search is not complete, but BA = 2.0 and DPHE = -.3927 radians appeared best and were utilized for the Mode 3 runs.

TABLE (7)

SPEED VARIATION

RUN #	HW=η (FT)	WL=λ (FT)	U=U (FT/SEC)	SI=ψ (RADS)	PHEW=φw (RADS)
1	7.0	280.0	90.0	0.0	-1.5708
2	7.0	280.0	85.0	0.0	-1.5708
3	7.0	280.0	80.0	0.0	-1.5708
4	7.0	280.0	75.0	0.0	-1.5708
5	7.0	280.0	70.0	0.0	-1.5708
6	7.0	280.0	65.0	0.0	-1.5708
7	7.0	280.0	60.0	0.0	-1.5708
8	7.0	280.0	55.0	0.0	-1.5708
9	7.0	280.0	50.0	0.0	-1.5708
10	7.0	280.0	45.0	0.0	-1.5708
11	7.0	280.0	40.0	0.0	-1.5708
12	7.0	280.0	35.0	0.0	-1.5708
13	7.0	280.0	30.0	0.0	-1.5708
14	7.0	280.0	25.0	0.0	-1.5708
15	7.0	280.0	25.0	3.1416	-1.5708
16	7.0	280.0	30.0	3.1416	-1.5708
17	7.0	280.0	35.0	3.1416	-1.5708
18	7.0	280.0	40.0	3.1416	-1.5708
19	7.0	280.0	45.0	3.1416	-1.5708
20	7.0	280.0	50.0	3.1416	-1.5708
21	7.0	280.0	55.0	3.1416	-1.5708
22	7.0	280.0	60.0	3.1416	-1.5708
23	7.0	280.0	65.0	3.1416	-1.5708
24	7.0	280.0	70.0	3.1416	-1.5708
25	7.0	280.0	75.0	3.1416	-1.5708
26	7.0	280.0	80.0	3.1416	-1.5708
27	7.0	280.0	85.0	3.1416	-1.5708
28	7.0	280.0	90.0	3.1416	-1.5708

TABLE (8)

WAVE LENGTH VARIATION

RUN #	HW=η (FT)	WL=λ (FT)	U=U (FT/SEC)	SI=ψ (RADS)	PHEW=ϕw (RADS)
1	3.5	140.0	80.0	0.0	-1.5708
2	4.2	168.0	80.0	0.0	-1.5708
3	4.9	196.0	80.0	0.0	-1.5708
4	5.6	224.0	80.0	0.0	-1.5708
5	6.3	252.0	80.0	0.0	-1.5708
6	7.0	280.0	80.0	0.0	-1.5708
7	7.0	280.0	80.0	3.1416	-1.5708
8	6.3	252.0	80.0	3.1416	-1.5708
9	5.6	224.0	80.0	3.1416	-1.5708
10	4.9	196.0	80.0	3.1416	-1.5708
11	4.2	168.0	80.0	3.1416	-1.5708
12	3.5	140.0	80.0	3.1416	-1.5708

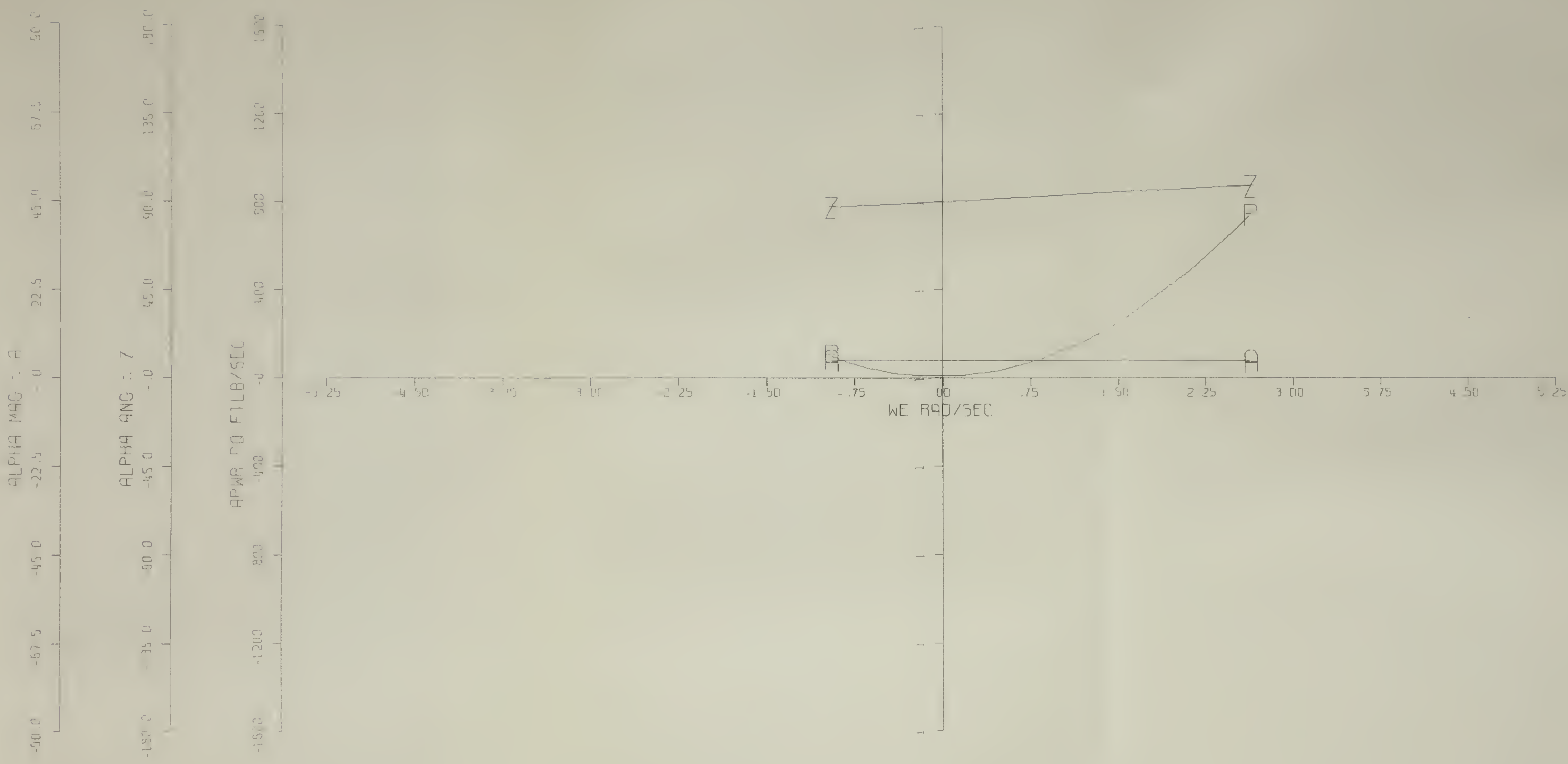

ALPHA MAG : A
ALPHA ANG : Z
APWR PO FTLB/SEC
WE RAD/SEC

figure 23
Full
Incidence
(SI) Var.

A = |a|
Z = ∠a
P = pavea

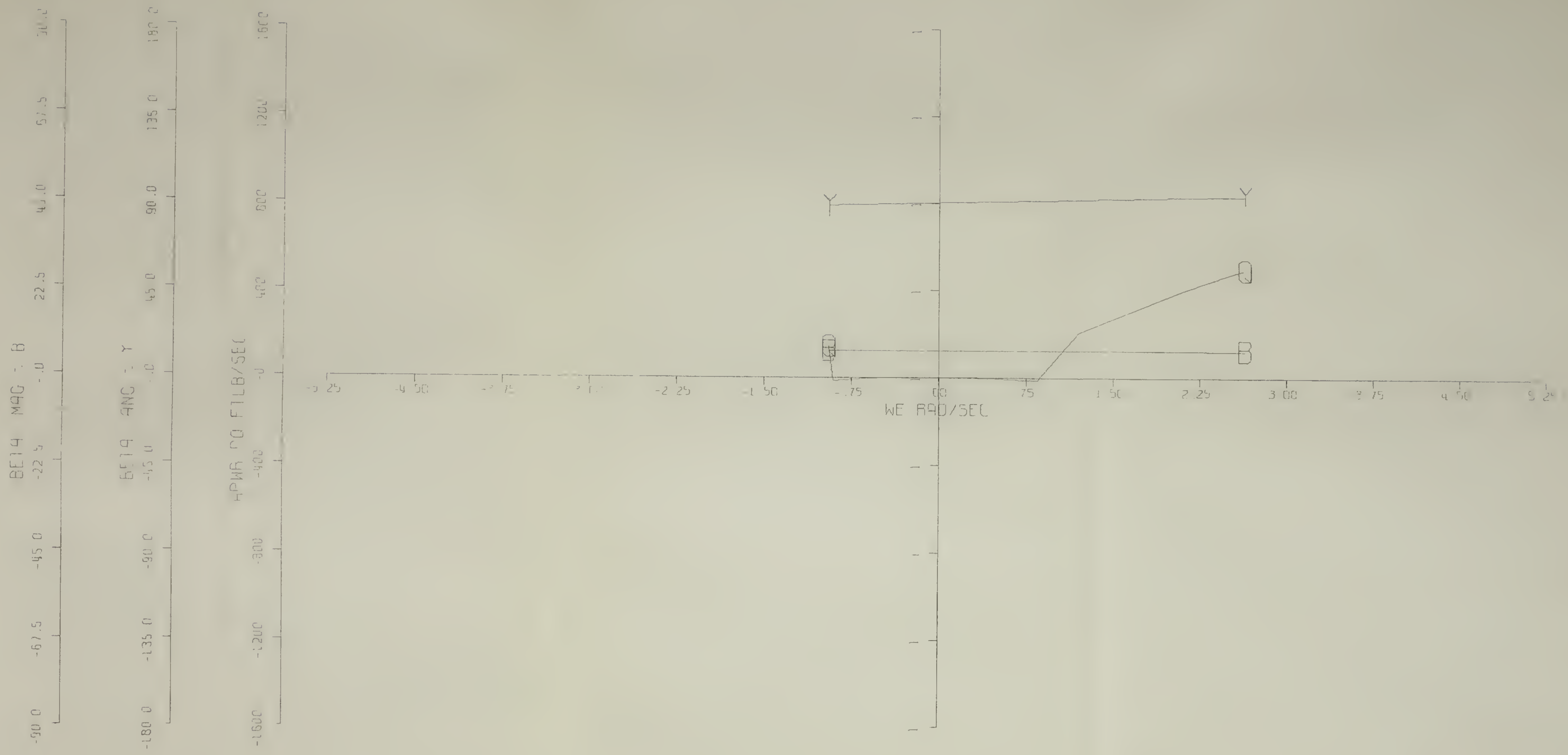

BETA MAG : B
BETA ANG : Y
HPWR PO FTLB/SEC
WE RAD/SEC

figure 24

Trailing Edge Flap (SI) Var.

B = $|\beta|$
Y = $L\beta$
Q = paveb

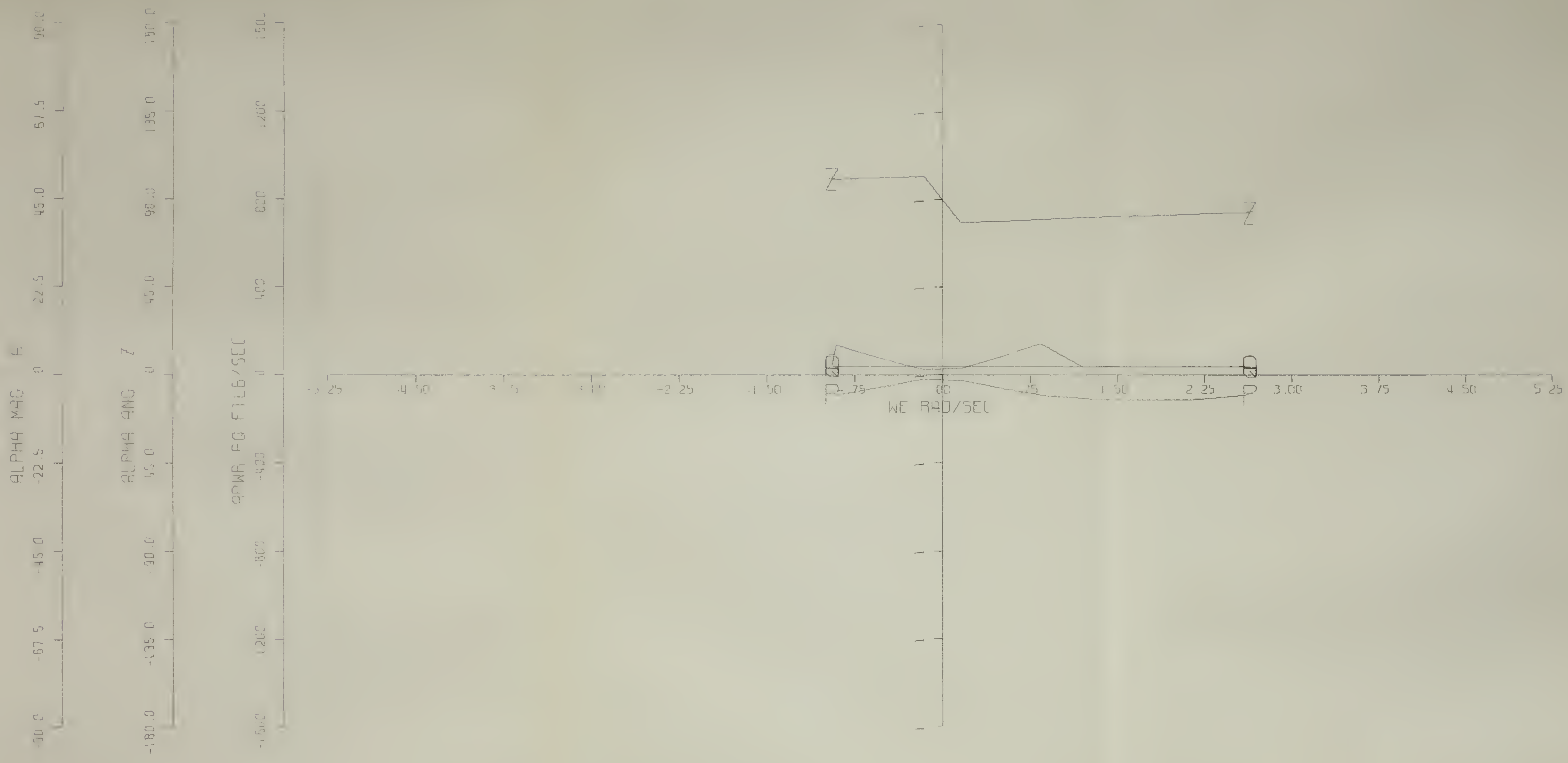

WE RAD/SEC
-4.50
-2.25
-1.50
.00
.75
1.50
2.25
3.00
3.75
4.50
5.25

figure 25
Tab
Foil
(SI) Var.

$A = |a|$
$|\beta| = 2 \cdot |a|$
$Z = \angle a$
$\angle\beta = \angle a - 22\frac{1}{2}^{\circ}$
$P = \text{pavea}$
$Q = -\text{paveb}$

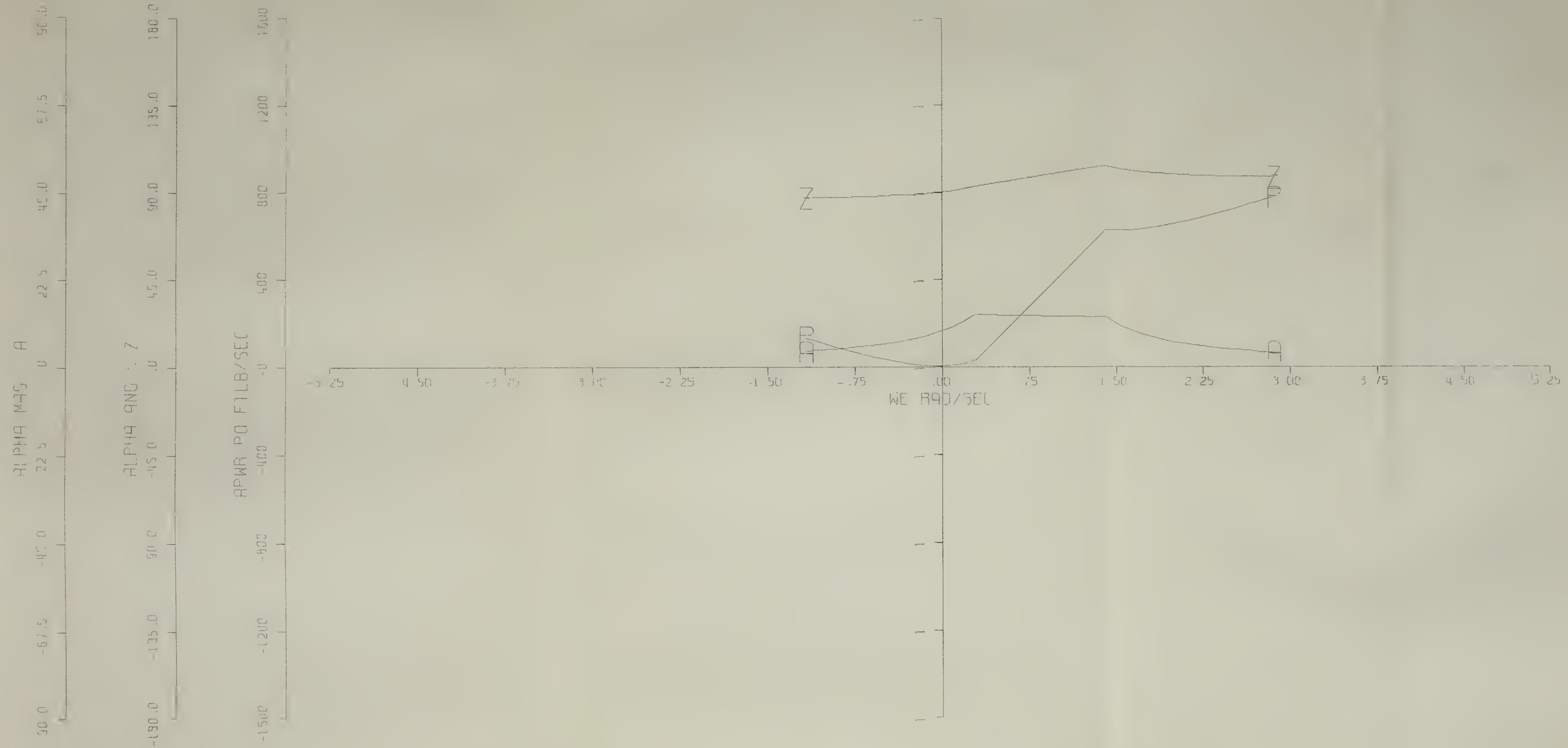
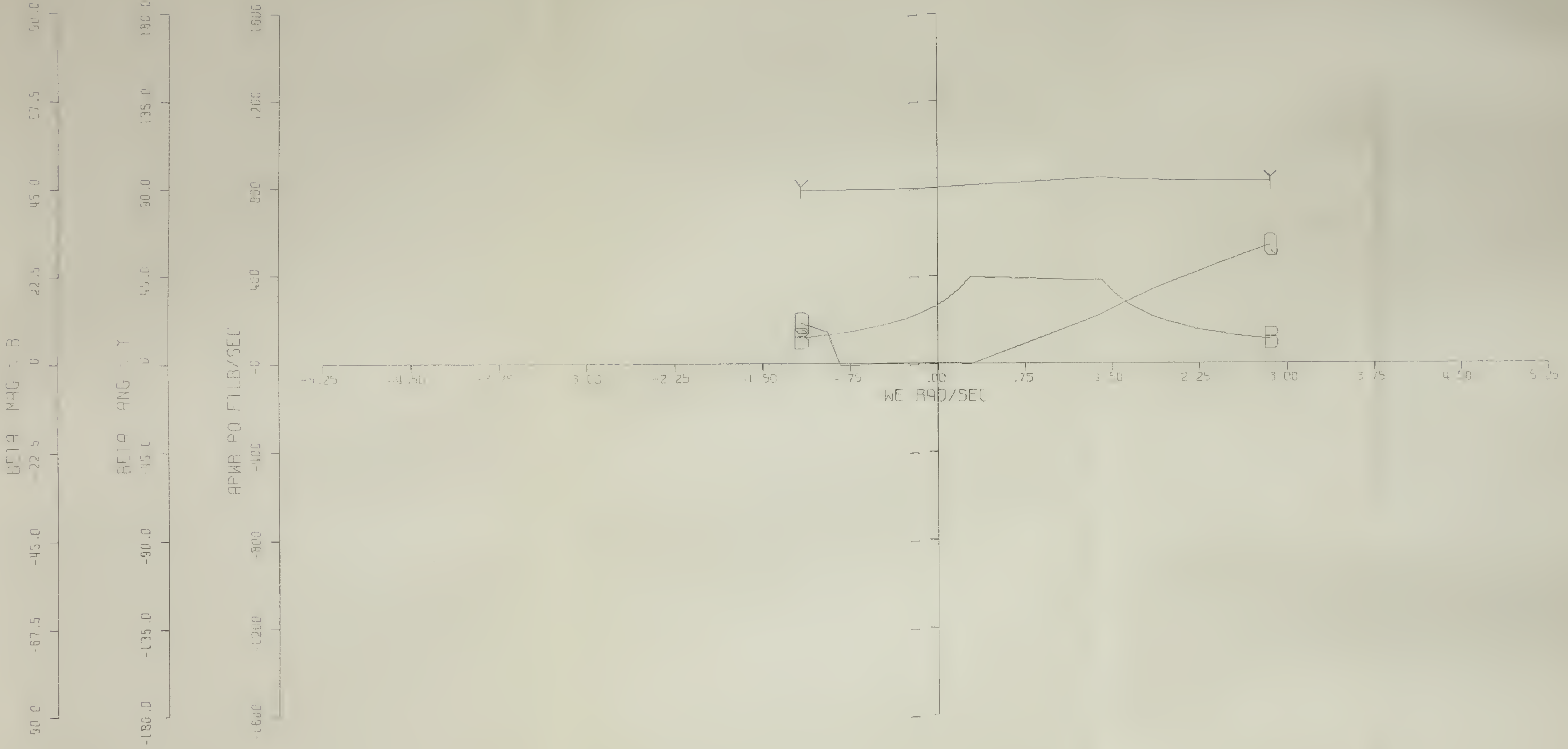

ALPHA MAG : A
ALPHA ANG : Z
APWR PO FTLB/SEC
WE RAD/SEC

figure 26
Full
Incidence
(U) Var.

A = $|\alpha|$
Ƶ = $\angle\alpha$
P = pavea



BETA MAG : B
BETA ANG : Y
APWR PO FTLB/SEC
WE RAD/SEC

figure 27
Trailing
Edge Flap
(U) Var.

B = $|\beta|$
Y = $\angle\beta$
Q = paveb

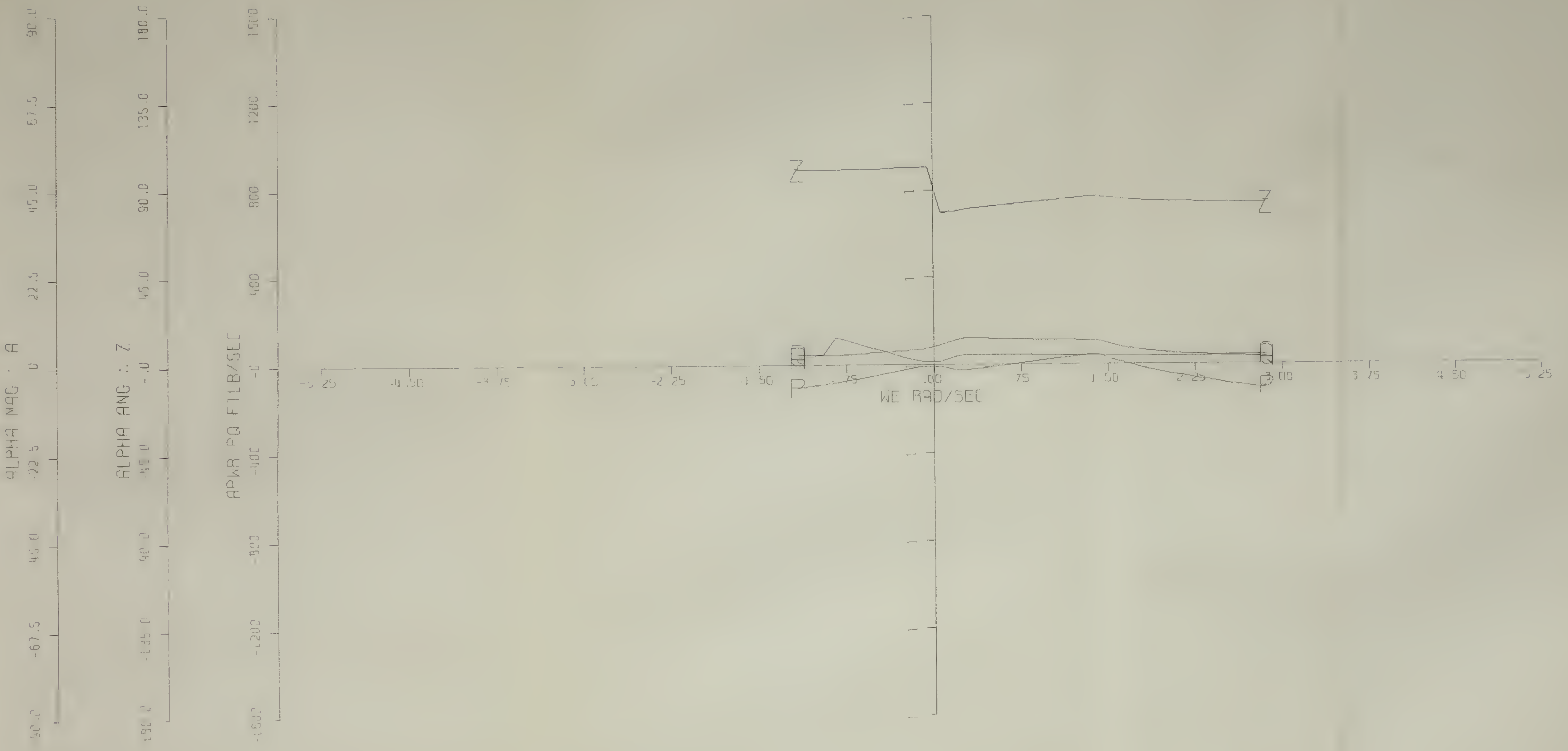

ALPHA MAG : A
ALPHA ANG : Z
APWR PQ FTLB/SEC
WE RAD/SEC

figure 28

Tab

Foil

(U) Var.

$A = |\alpha|$

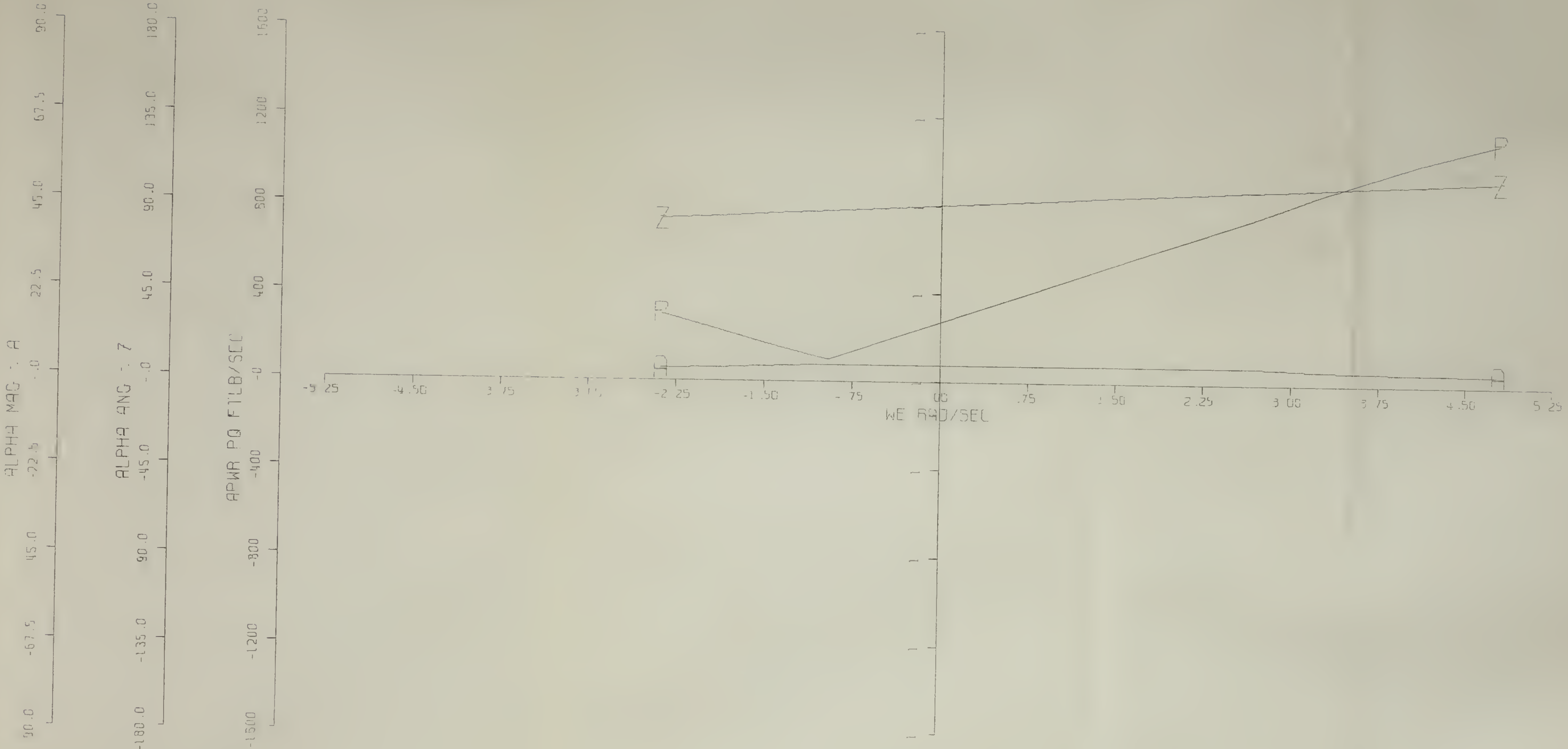
$|\beta| = 2 \cdot |\alpha|$

$Z = \angle\alpha$

$\angle\beta = \angle\alpha - 22\tfrac{1}{2}^\circ$

$P = \text{pavea}$

$Q = -\text{paveb}$


ALPHA MAG : A
ALPHA ANG : Z
APWR PQ FTLB/SEC
WE RAD/SEC

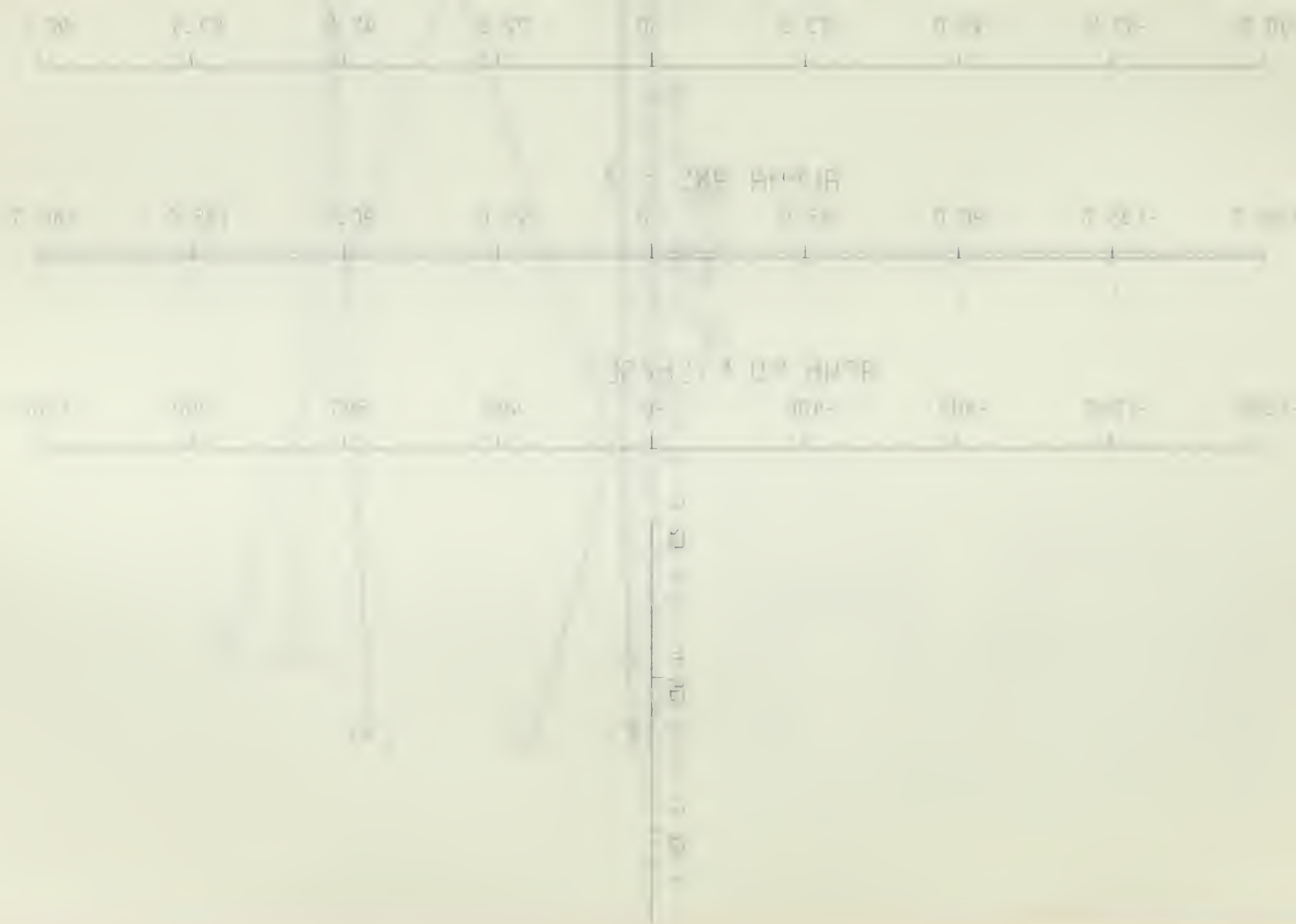


figure 29
Full
Incidence
(WL) Var.

$A = |\alpha|$
$Z = \angle\alpha$
P = pavea

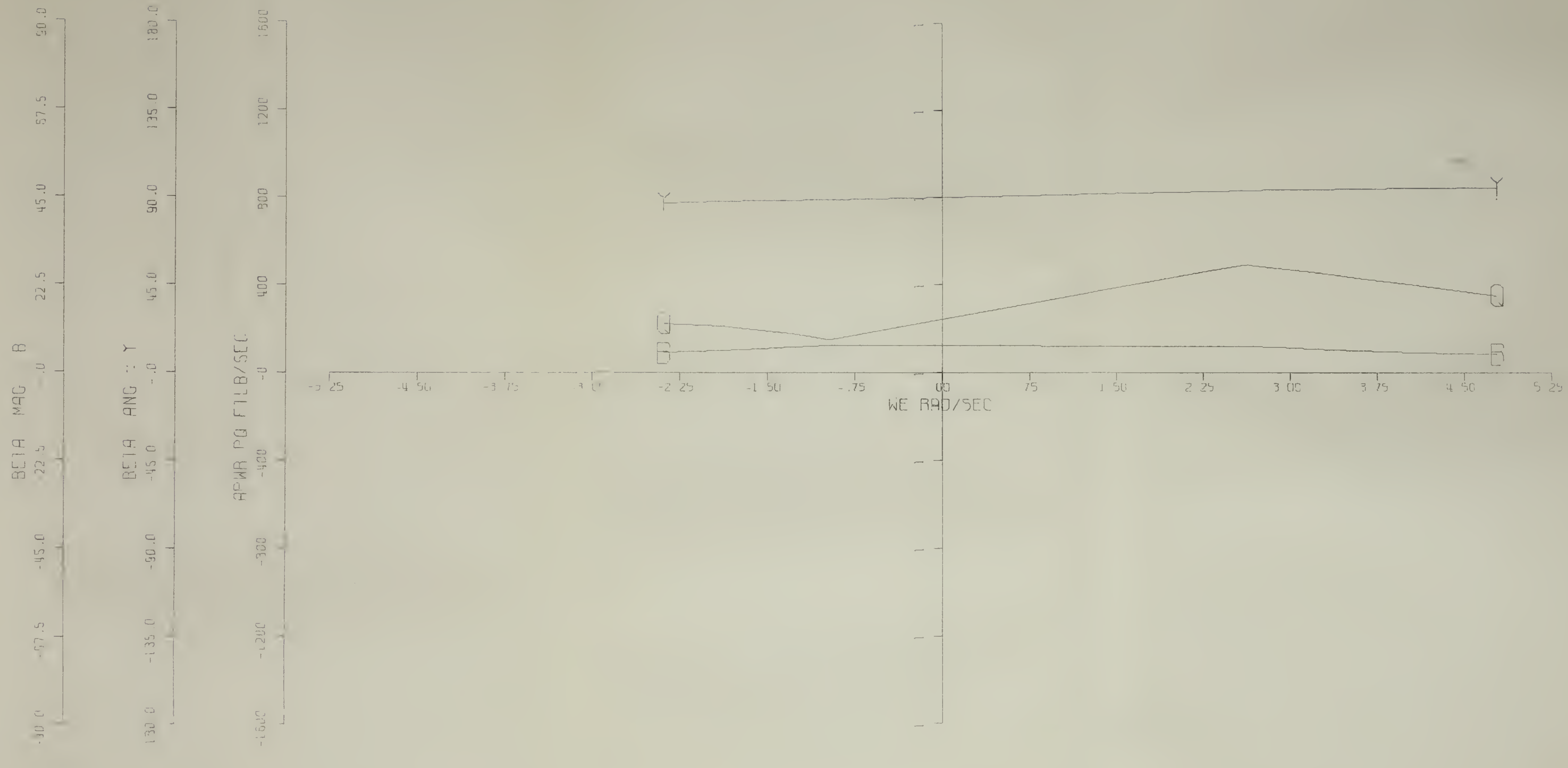

BETA MAG B
BETA ANG : Y
APWR PG FTLB/SEC
WE RAD/SEC

figure 30
Trailing
Edge Flap
(WL) Var.

B = $|\beta|$
Y = $\angle\beta$
Q = paveb

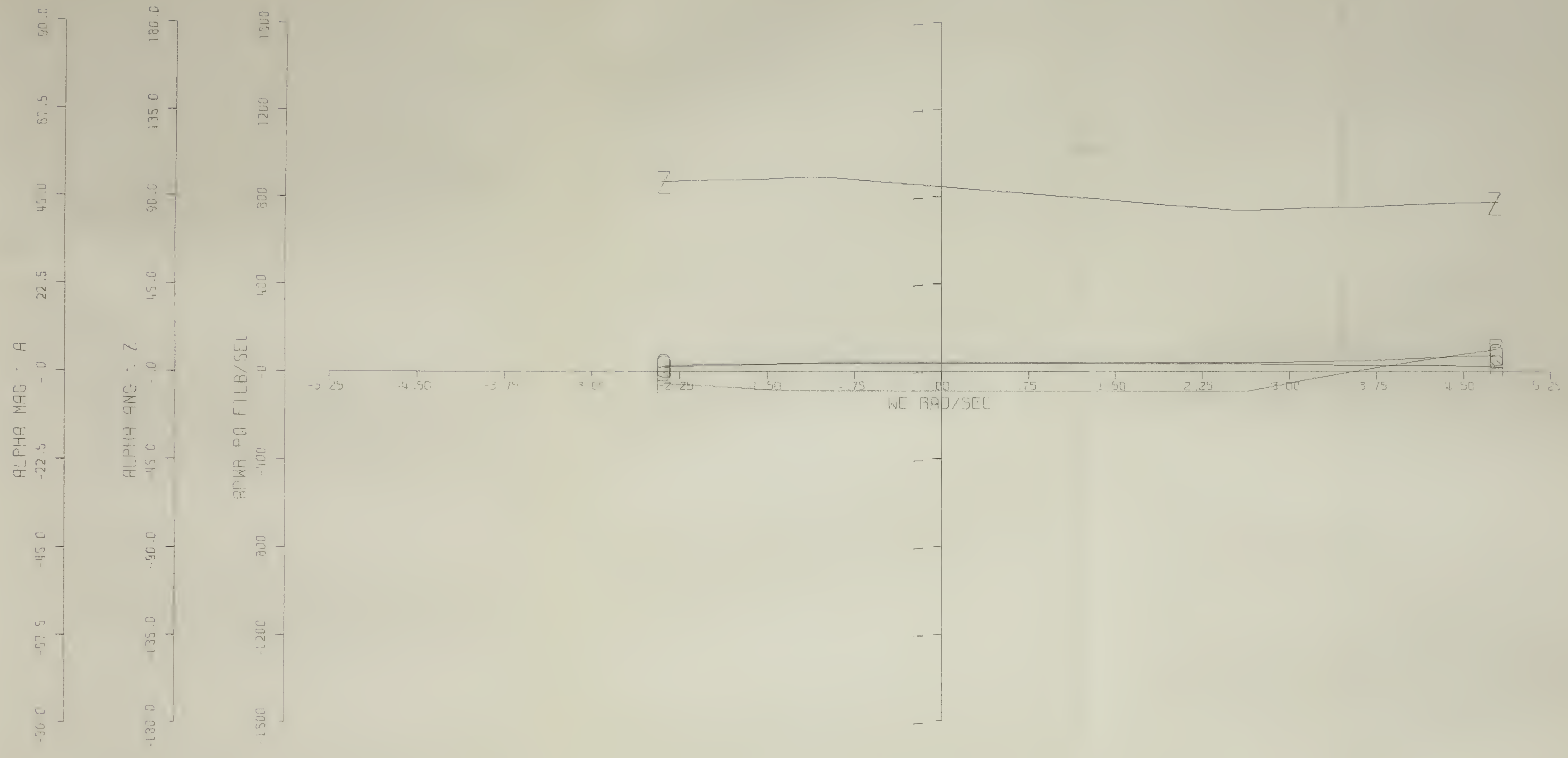

ALPHA MAG : A
ALPHA ANG : Z
APWR PO FTLB/SEC
WC RAD/SEC

figure 31
Tab
Foil
(WL) Var.

$A = |a|$
$|\beta| = 2 \cdot |a|$
$Z = \angle a$
$\angle\beta = \angle a - 22\frac{1}{2}^{\circ}$
$P = \text{pavea}$
$Q = -\ \text{paveb}$

All the results are in error for the range of reduced frequency, $-.05<k<+.05$, because of the fix-up procedure mentioned in Section H. The error is dramatic because of the sensitivity of power to the related phase angles of its components. The error is zero at $k = 0$, and beyond the range $k = \pm.05$. The range for k translates into a different range of ω_e for each of the three wave series. The reason is that k is composed of both ω_e and U, which are not solely functions of one another, Equation (27). For a given U, ω_e remains dependent on λ and ψ. The following are the ranges of ω_e which are in error for:

Series 1. (SI) Variation

$-.85<\omega_e<+.85$ (Rads/Sec)

Series 2. (U) Variation

$-.94<\omega_e<+.29$ (Rads/Sec)

Series 3. (WL) Variation

Runs not in the range

The plotted results are somewhat misleading in a part of the ω_e range, because the plotter plots straight lines between points. The following are the ranges in ω_e where there are significant gaps in plotted points:

1. (SI) Variation

None

2. (U) Variation

$$+.28<\omega_e<+1.41 \quad \text{(Rads/Sec)}$$

3. (WL) Variation

$$-.94<\omega_e<+2.64 \quad \text{(Rads/Sec)}$$

Aside from the fix-up error and taking note of the range for the gap in points, the plotted results give a reasonably good picture of the power required.

In the case of Mode 3 $\overline{\text{ALPHA}}$ and $\overline{\text{BETA}}$ Motion (Tab Foil), the plotter plots the Beta power as Q= - PAVEB. This operation places the Q result relative to the P result (PAVEA) such that the distance between them is the total power required by the system. If the distance between Q → P is in the <u>positive</u> direction, the total power required is <u>positive</u>. If the distance between Q → P is in the <u>negative</u> direction, the total power required is <u>negative</u>. The above is true regardless of where the plots lie on the absolute scale.

THE FOLLOWING ARE COMMENTS ON EACH <u>FOIL</u> RESULT:

1. Alpha Motion (Full Incidence) Mode 1

a. (SI) Variation. Figure (23) shows a decreasing average power requirement from "dead ahead" around to quartering seas where the encounter frequency goes to zero (power goes to zero); and then continuing around to "dead astern", the power increases again for ω_e = (-). The maximum power occurs at SI = 0 and it is positive.

The magnitude of the Alpha motion is constant, because the disturbance is unchanged in magnitude and the ship's (lift generating) speed is constant. Note, at $\omega_e = 0$ (SI ≈ 112°), the magnitude of Alpha does not go to zero. This is because, depending upon what the phase angle of the wave is that the ship is running along (crest, back-face, trough, or front-face), Alpha will have to be a constant 0, (+), 0, (-).

The phase angle of Alpha relative to the wave height changes from lagging 90° + ε for ω_e = (+), to leading 90° - ε for ω_e = (-). At $\omega_e = 0$, the phase angle is 90° from the wave height; but, more important, it is 180° out of phase with the wave upwash velocity $\overline{UPW}$ as is expected from quasi-steady analysis. The basic characteristic of shifting from lagging to leading and being out of phase with $\overline{UPW}$ at $\omega_e = 0$ is common to all the foils in all the wave series.

b. (U) Variation. Figure (26) shows increasing magnitude for the required Alpha motion, as speed drops from 90 ft/sec in ahead seas (lesser ω_e = (+)). The magnitude decreases as the ship speed increases in astern seas (greater ω_e = (-)).

The phase angle goes through a similar variation as in the SI series.

The average power decreases as the ship slows in ahead seas. In astern seas, the power dips down through zero at $\omega_e = 0$ and then climbs back up as speed increases. The gap

between points at ± 25 ft/sec is necessary because of the limited capability of the lifting foils to support the ship at low speed. This is evident by the rapidly increasing motion magnitude (shown in Figure (26)) required to produce the lift at low speeds.

c. (WL) Variation. Figure (29) shows the magnitude of Alpha motion increases slightly as the wave length increases in ahead seas. This is because the increased wave length presents an increasing disturbance. The same variation appears in astern seas.

The phase angle changes in a similar fashion as in the SI series.

The average power <u>decreases</u> as wave length <u>increases</u> in ahead seas (lesser ω_e = (+)). In astern seas, the power again <u>decreases</u> as wave length <u>increases</u> (lesser ω_e = (-)). Note, there is no effect of $\overline{C(k)}$ error in this plot, because ω_e is out of the range.

2. Beta Motion (Trailing Edge Flap) Mode 2

a. (SI) Variation. Figure (24) shows the magnitude of the Beta motion constant as in the Alpha case. The magnitude of Beta is larger than the Alpha case which is to be expected for a 1/4 chord flap compared to full incidence.

The phase angle goes through a similar variation as in the Alpha case.

b. (U) Variation. Figure (27) shows the magnitude increasing in a similar manner as in the Alpha case. Again, the magnitude of Beta is larger than that for Alpha.

The phase angle change is similar to the (SI) variation.

The average power follows a similar trend as in the Alpha case; but, again, it's sensitivity to error in $\overline{C(k)}$, for the range $-.94<\omega_e<+.29$, is much greater. The only point that is correct in this range is at $\omega_e = 0$. Again, it is speculated that, if correct values for $\overline{C(k)}$ were used, the power curve would fair into zero at $\omega_e = 0$ and climb back up again in a similar fashion to the Alpha case.

c. (WL) Variation. Figure (30) shows the magnitude of Beta increasing in a similar fashion as in the Alpha case.

The phase angle changes in the same manner as in the Alpha case.

The average power <u>increases</u> for <u>increasing</u> wave length in ahead seas (lesser $\omega_e = (+)$). This result is in contrast with the Alpha result where power decreased for increasing wave length in ahead seas. The average power in astern seas <u>decreases</u> for increasing wave length (lesser $\omega_e = (-)$). Note, there is no effect of $\overline{C(k)}$ error in this plot, because ω_e is out of the range.

3. Alpha and Beta Motion (Tab Foil) Mode 3

a. (SI) Variation. Figure (25) shows the magnitude of Alpha as a constant straight line as in the Alpha only

case. The associated Beta magnitude is two times the Alpha magnitude; however, it is not plotted. This is true for the (U) variation and (WL) variation as well.

The Alpha phase angle has a similar trend, as before, but the $\omega_e = (\pm)$ variations are displaced away from 90°. The Alpha motion now lags the wave height by $90° - \varepsilon$ for $\omega_e = (+)$, and leads by $90° + \varepsilon$ for $\omega_e = (-)$. The ramp-type jog near $\omega_e = 0$ is caused by the plotter moving from $\omega_e = +.16 \rightarrow -.14$. The jog would approach a step type for points approaching $\omega_e = 0$. The Beta phase angle lags the Alpha phase angle by 22 1/2° for both $\omega_e = (\pm)$; however, it is not plotted. This is true for the (U) variation and (WL) variation as well.

The results for power are important. As was pointed out earlier in this section; if the distance between Q (Beta power) → P (Alpha power) is in the <u>negative</u> direction, the total power to drive the motion is <u>negative</u> (the control system must absorb power from the motion). Figure (25)'s plot of Q, P is exactly this case. Figure (32) demonstrates in qualitative form the relative position of the motion phasors and their lift phasors.

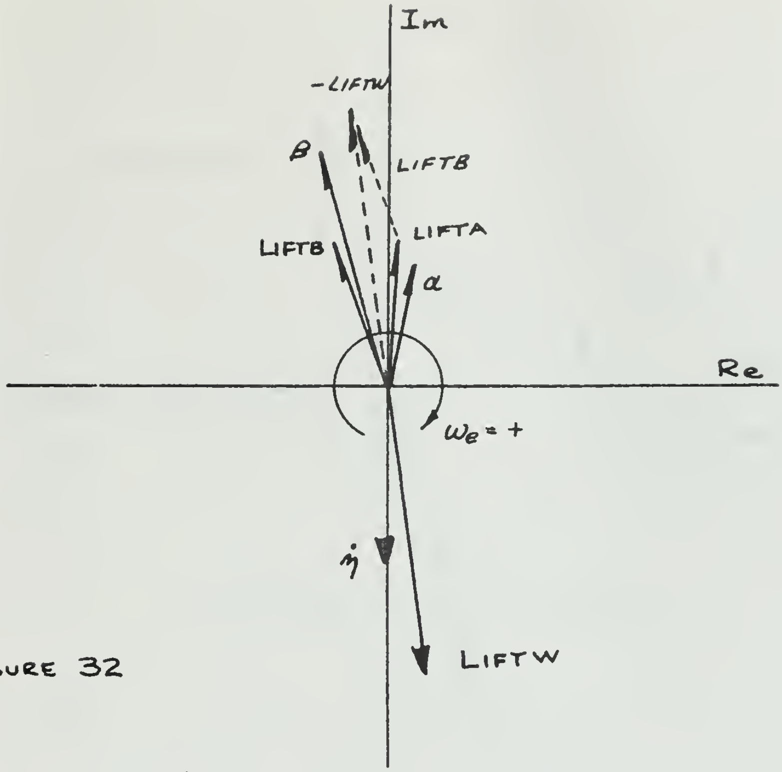


FIGURE 32

Figures (33) and (34) demonstrate in qualitative form how these phasors contribute to <u>negative</u> power.

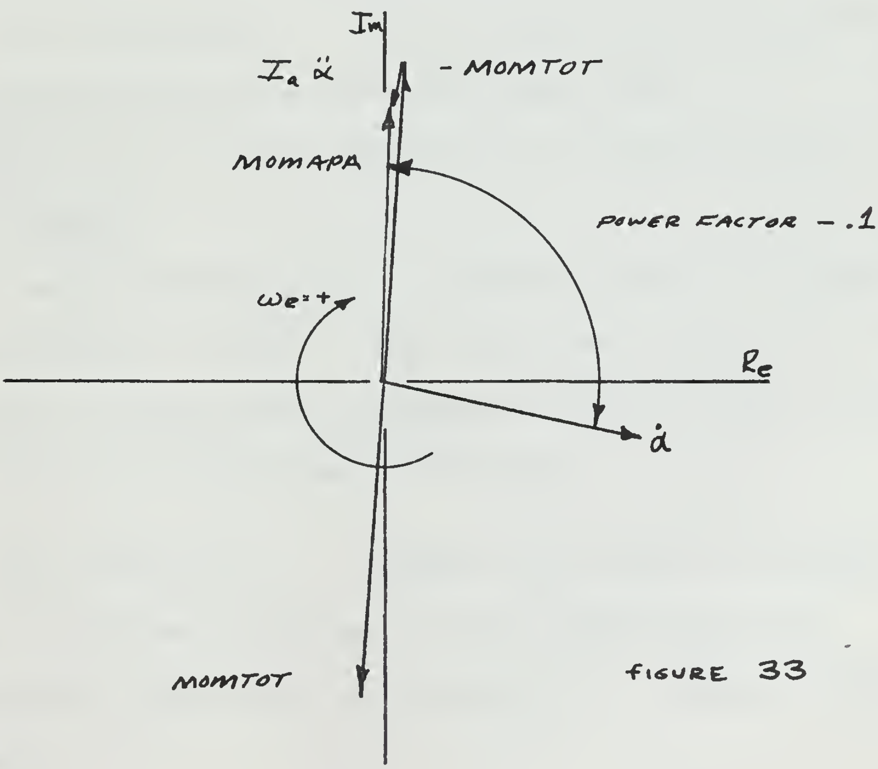


FIGURE 33

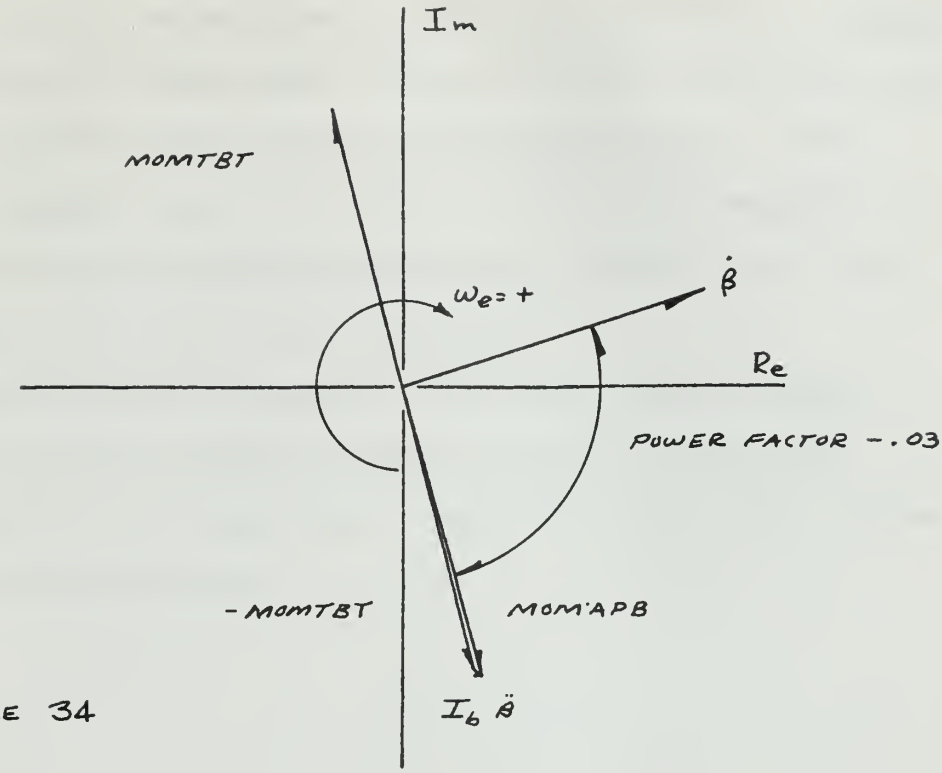


FIGURE 34

b. (U) Variation. Figure (28) shows the magnitude of Alpha increasing as in the Alpha only case.

The Alpha phase angle variation is similar to the Mode 3 SI case.

The average total power is negative over the range of ω_e except near $\omega_e = +1.50$. Here the power required by Alpha is positive, but the power required by Beta is just enough negative for the total power to be zero.

c. (WL) Variation. Figure (31) shows a slightly increasing Alpha magnitude as in the Alpha only case.

The Alpha phase angle is changing in a manner similar to the SI case.

The total average power starts out positive for <u>small</u> wave lengths in ahead seas. At $\omega_e = +4.28$, the total power is zero and then goes negative for decreasing ω_e. The average negative power for Beta is not enough negative to cancel the positive Alpha power for ω_e +4.28. Note, the $\overline{C(k)}$ error does not effect this plot.

From the above results, it is clear that an understanding of the unsteady effects on foils is not only necessary but also rewarding. Table (9) lists some of the important power results.

TABLE (9)

	(Full Incidence) Alpha Motion Maximum ± Average Power ft-lb/sec per ft span	(Trailing Edge Flap) Beta Motion Maximum ± Average Power ft-lb/sec per ft span	(Tab Foil) Alpha and Beta Motion Maximum ± Average Power ft-lb/sec per ft span
SI	+ 738.5	+ 468.0	- 154.9
U	+ 774.5	+ 534.7	- 153.5
WL	+1105.2	+491.2	+ 23.6, - 133.8

V. DRAG

The components of drag for a foil are:

Skin Friction

Pressure Drag

Induced Drag

Wave Drag

Interference Drag

Each of these has a contribution from:

Steady Motion

Waves

Foil Motion

The task of sorting out all of these parts is formidable. For a 2-D analysis, the induced parts can be eliminated. For a small strut, the interference drag can be neglected. For a well-submerged foil, the wave drag can be neglected. Assuming small motions with no separation, cavitation or ventilation, the pressure drag can be neglected.

This leaves the effects of steady motion, waves, and foil motion on skin-friction drag. The temptation is to linearly superpose these three effects as was done for lift. Yet the model for skin friction is the boundary layer.

Reference (2) describes the effect on foil-skin friction by a uniform inflow with a harmonic perturbation superimposed. This is a model of the effect of waves on the skin friction of a steady foil. The shearing stress leads the velocity perturbation (horizontal orbital velocity for the wave), and the phase angle approaches 45° for $k \to \infty$.

An oscillating cylinder in a fluid at rest is discussed, and an oscillating infinite flat plate in a fluid at rest is discussed. The problem of a finite chord oscillating flat-plate foil in uniform flow is not discussed. No mention is made of the advisability of superposing such a solution on the effects due to waves.

Although the required motion of the control surface results in zero total lift, the lift distribution is different for the wave and for the motion. This is evidenced by the different values for wave moments and motion moments. Therefore, the pressure distribution will oscillate back and forth across the foil. It is speculated that the problem can no longer be solved by linear superposition, even if there was a solution to the foil motion effects. The solution of the coupled effects of waves and motion appears beyond analytical solution today.

All that is left is skin-friction drag from steady motion in uniform flow. The result for the three candidates, with the same overall foil geometry, is that there is no significant difference in skin-friction drag.

This statement is adequate for comparing the overall power of the three candidates. However, if comparisons of these foils were being made with other systems, such as spoilers, the above rationalization is not adequate because ventilation and separation play a much different part in the drag of such systems.

VI. WEIGHT

Any statement about weight without a first iteration of the complete control system design must be qualitative. However, for the effect of weight to influence the choice of control system, an absolute figure must be found. For the purpose of emphasizing that power must be supplied to lift the weight of the control system, an approximate analysis will be pursued.

A. REFERENCE FULL-INCIDENCE SYSTEM

To use as a reference system, the weight of a full-incidence (Alpha motion) control system is estimated as follows:

Group I	Foil Weight	=	15,700 lbs	≈ 740 lbs/ft
Group II	Mechanical Drive Links	=	650 lbs	≈ 30 lbs/ft
	Hydraulic Servo Cylinders	=	650 lbs	≈ 30 lbs/ft
	Hydraulic Fluids and Pipes	=	1,070 lbs	≈ 50 lbs/ft
	Hydraulic Pumps and Drive	=	400 lbs	≈ 20 lbs/ft
Group III	Autopilot and Computer	=	200 lbs	≈ 10 lbs/ft
Group IV	Sensors	=	200 lbs	≈ 10 lbs/ft

B. COMPARATIVE WEIGHT

The Group I, III, and IV weights for the three candidates will not be significantly different. The Group II weights

will approximately vary linearly with average power. For comparison, from Table (9):

$$\frac{\text{Trailing Edge Flap Max Power}}{\text{Full Incidence Max Power}} = \frac{534.7}{1105.2} \approx .48$$

$$\frac{\text{Tab Foil Max Power}}{\text{Full Incidence Max Power}} = \frac{-154.9}{1105.2} \approx .14$$

(Here the maximum negative power is taken to size the servo machinery even though it must absorb the power.)

C. TRAILING EDGE FLAP SYSTEM WEIGHT

Group II $\approx$ 70 lbs/ft

D. TAB FOIL SYSTEM WEIGHT

Group II $\approx$ 20 lbs/ft

E. POWER

The weight becomes power through the lift/drag ratio and the speed of the hydrofoil ship:

$$\text{Power} = (\text{Drag/Lift})\cdot\text{Weight}\cdot\text{Speed} \qquad (59)$$

For comparative purposes, the lift/drag ratio is 7 and the speed is 80 ft/sec. Power results due to weight are as follows:

Full Incidence	$\approx$	1500 ft-lb/sec per ft span
Trailing Edge Flap	$\approx$	800 ft-lb/sec per ft span
Tab Foil	$\approx$	230 ft-lb/sec per ft span

VII. CONCLUSIONS AND RECOMMENDATIONS

In summary, the contributions of power, power due to drag, and power due to weight have been developed for three candidate hydrodynamic devices for use in a constant lift, wave alleviation, control system for a hydrofoil ship.

A. OVERALL CONTROL SYSTEM POWER

A servo system efficiency is required to convert the control surface power into input power to the system. An efficiency of .6 is assumed and has been applied to the maximum power results in Table (9).

Table (10) shows the results for overall control system power.

TABLE (10)

	Full Incidence ft-lb/sec per ft span	Trailing Edge Flap ft-lb/sec per ft span	Tab Foil ft-lb/sec per ft span
Power	+ 1840.	+ 890.	+ 40.
Drag Power	—	—	—
Weight Power	+ 1500.	+ 800.	+ 230.
Total	+ 3340.	+ 1690.	+ 270.

From this analysis, the tab foil looks very promising for use as the hydrodynamic device in a constant lift control system. Development of the sensor, servo, and autopilot system designs for use with the tab foil is recommended.

If the choice is restricted to either full incidence or trailing edge flap, the trailing edge flap looks most promising. The development of a control system utilizing a full incidence foil for stability and maneuverability, and a trailing edge flap for wave alleviation, is recommended.

But the most important recommendations are:

1. Wave alleviation should be an important consideration in the design of an open-ocean hydrofoil ship.
2. Unsteady lift analysis should be used during the design of the control system for an open-ocean hydrofoil ship.

B. SENSORS

The control system must be able to sense the magnitude of the upwash velocity, the direction, and the encounter frequency of the wave the foil is about to see, enough in advance, in order to control proper foil motion. Wave height information is not enough. It is suggested that wave height and orbital velocity be measured ahead and behind the ship.

C. MOTION

Figure (35) shows full incidence, trailing edge flap, and tab foil motion. The amplitudes have been exaggerated,

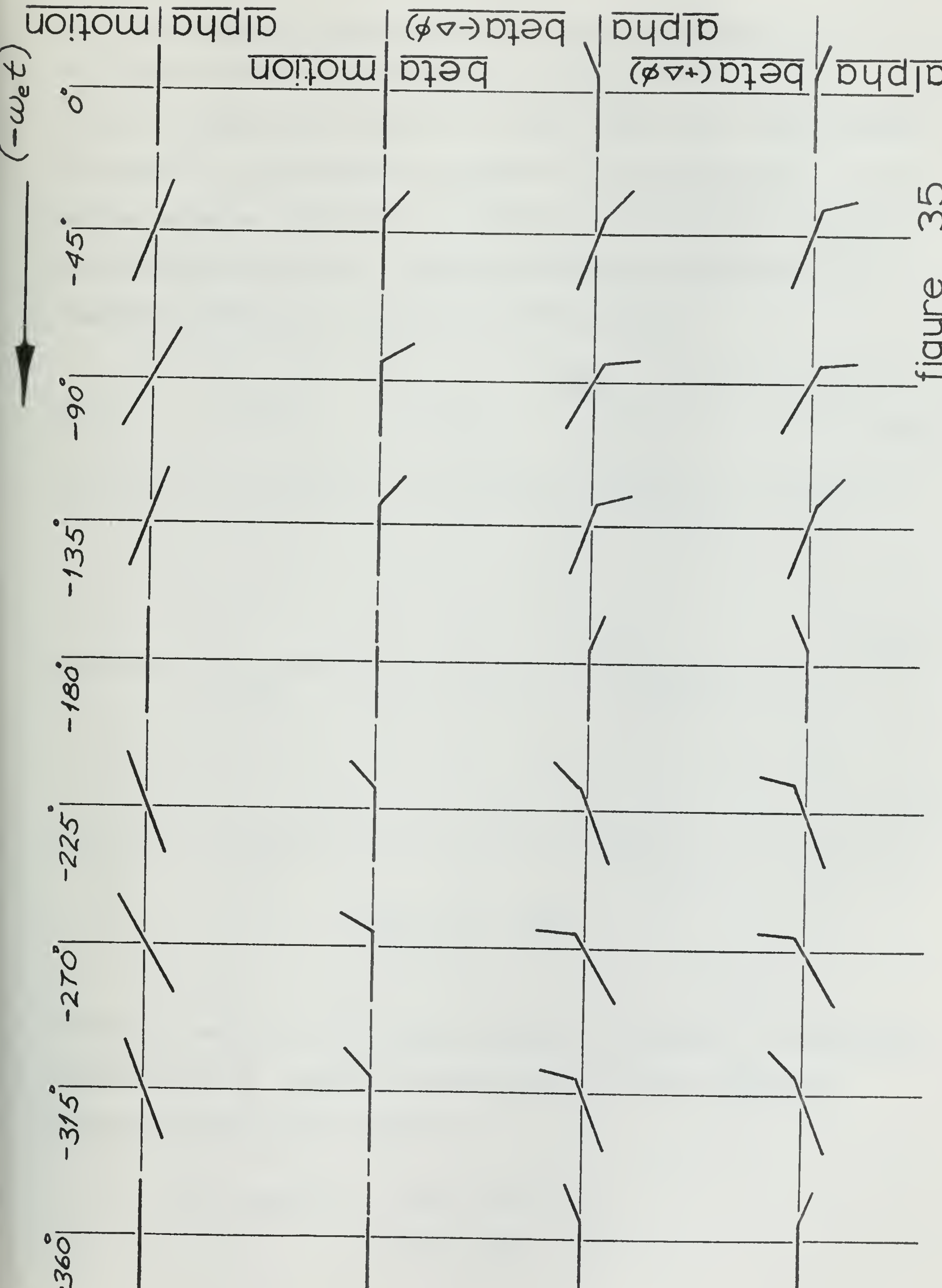

$(-\omega_e t)$
0°
-45°
-90°
-135°
-180°
-225°
-270°
-315°
-360°
alpha motion
beta motion
beta(-Δϕ)
alpha
beta(+Δϕ)
alpha

figure 35

but their relative magnitudes have been maintained. The most interesting motion is that of the tab foil. With 22 1/2° lagging phase angle for Beta ($-\Delta\phi$), the Alpha motion is caused by the moments generated by the Beta motion. The motion appears "natural" in contrast to the 22 1/2° leading system shown below it. Figure (36) shows motions for 90° lagging, 180° out of phase, and 90° leading.

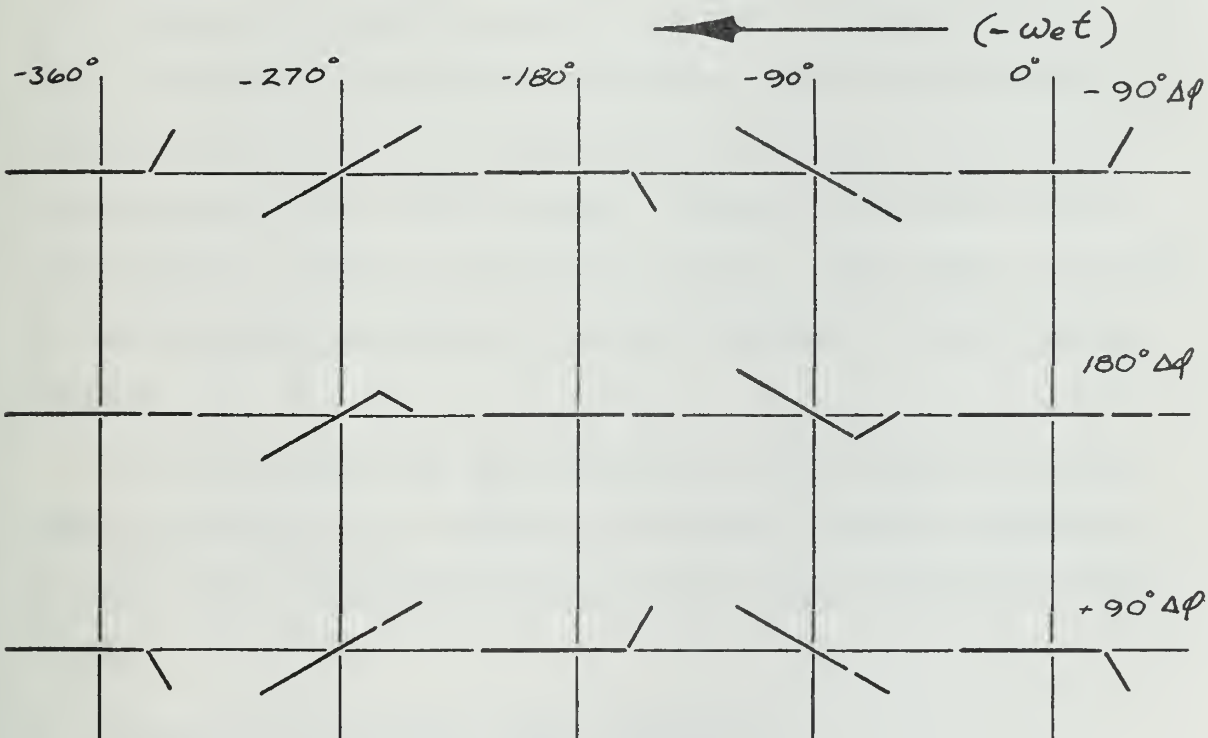


figure 36

Of these, only the 90° lagging appears "natural". Preliminary checks with the computer program for 80 ft/sec speed in limiting ahead seas, indicate:

90° Lagging } Alpha Power << 0

Beta Power << 0

180° Out of Phase } 0 << Alpha Power

Beta Power < 0

90° Leading } 0 << Alpha Power

0 << Beta Power

The "natural" looking motions require negative power. The other motions require positive power. References (3) and (4) discuss the use of a tab to initiate aileron motion. The tab is moved in the opposite direction from the direction the aileron will move. At first, this sounds like the aileron tab should be 180° out of phase. However, the Beta motion must lead (in time) to cause the motion. This lead (in time) in the opposite direction is a lag relative to the aileron motion.

For the hydrofoil ship, lags near 90° result in large negative powers to be absorbed (like the flutter situation). If small lags around 22 1/2° are used, small negative powers result.

D. EFFECTS ON HYDROFOIL SHIP OPERATION

In the limiting sea (ahead or astern), slowing down the ship will reduce the control system power but will increase the magnitude of the control surface motion. Since most foil arrangements have a mechanical limit to foil travel, slowing down may not be the best operational change. Running with the wave requires no foil motion and no control system power. Running with the crest or trough requires no control

surface angle. Running with the face or back requires full - or + control surface angle. Running with the crest is recommended because no control surface angle is required and maximum submergence of the foils is obtained.

The angle of attack to the sea will reduce the control system power but not the foil motion magnitude. When running along the wave, the face or back of the wave present no spanwise flow; but the control surface will be at full - or + angle. Running along the crest or trough will present a small spanwise flow to the foils and struts, but no control surface angle will be required. Running along the trough is recommended because no control surface angle is required and maximum submergence of the foils, on the beam, is obtained.

Finding smaller waves decreases control surface motion in all cases, and will reduce the control system power for a trailing edge flap system in ahead seas. But, smaller waves increase control system power for the full incidence system in both ahead and astern seas. Smaller waves increase control system power for the trailing edge flap system in astern seas.

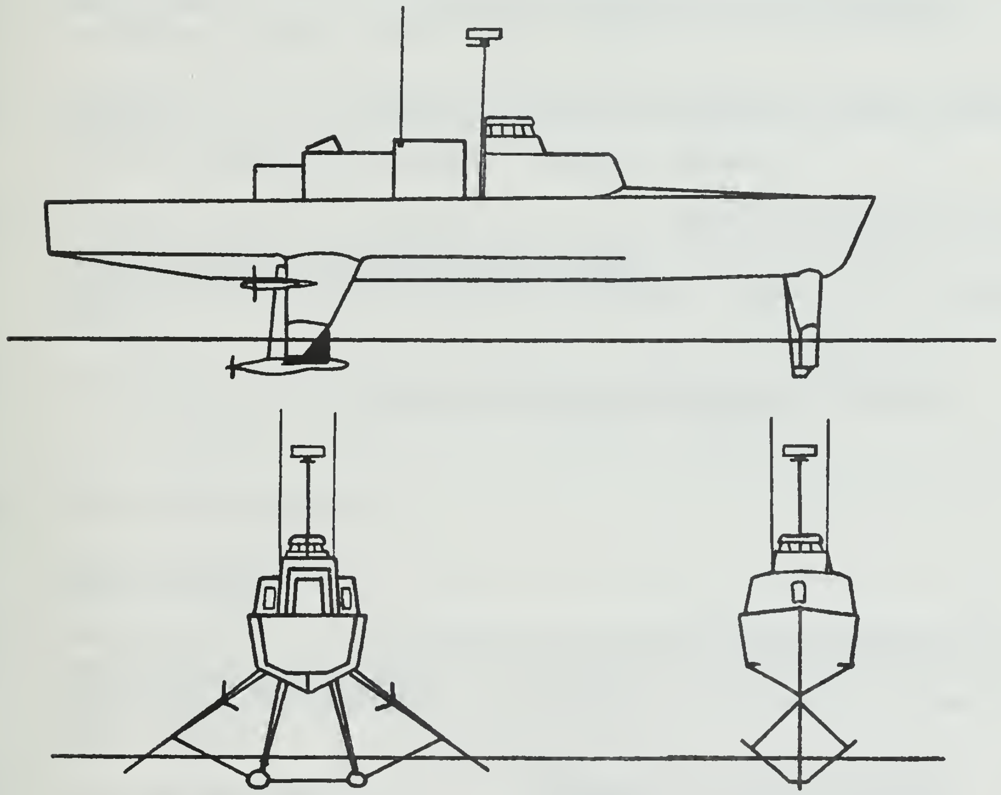
It is recommended that wave alleviation unsteady effects be analyzed before determining operational policy.

E. STATISTICAL ANALYSIS

The foil responds to wave upwash velocity as an input. Therefore, an orbital velocity spectrum with direction information is required. This is a 3-dimensional-surface from

statistical wave data. Correspondingly, the foil response spectrum is no longer a single curve but is a 3-dimensional-surface <u>for a given ship's speed</u>. The complete picture is a series of response surfaces for a range of speeds.

It is recommended that an attempt be made at bringing the unsteady foil response analysis into a statistical format. From this, a complete and realistic description of hydrofoil ship energy requirements could be made.



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APPENDIX A: HYDRODYNAMIC DEVICES FOR SECTION III-A

I. ANGLE OF ATTACK CHANGERS

A. TRANSLATING FOIL (SUSPENDED FOIL)

The foil vertical velocity must match that of the incoming wave. If the speed of the foil is zero, $\omega_e = \omega_w$. The vertical velocity of the incoming wave will be $|\dot{\eta}| = \eta\omega_w$. The velocity of the foil must be $|\dot{z}| = |\dot{\eta}|$; but, if $\omega_e = \omega_w$, than $|z| = |\eta|$. In other words, the foil motion would exactly follow the wave surface. If speed is increased, the magnitude of ω_e increases and $|z| < |\eta|$. Such large excursions would probably not be feasible. A patent for a similar scheme is in Reference (5).

II. CAMBER CHANGERS

A. PARTIAL CHORD FLUID (JET FLAP)

Much work has been done on jet flaps for high lift on aircraft. The feasibility of pumping water through the foil has not been determined as yet. Fluidic controls could be employed to change the direction of the jet for wave alleviation. See References (6) and (7).

B. UPPER SURFACE MECHANICAL (MEMBRANE)

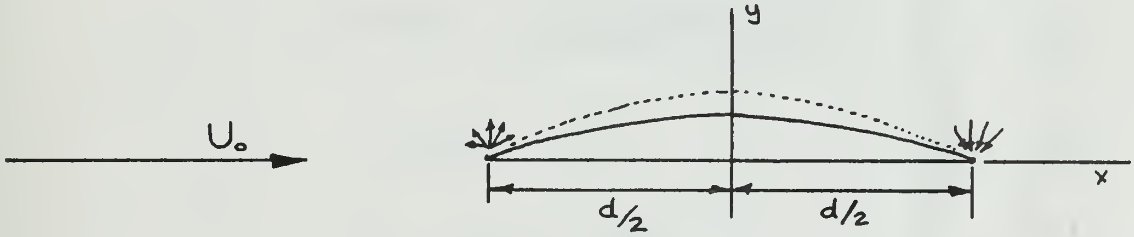
This scheme would employ a second skin for the upper surface of the foil. The space between could be pressurized to change the membrane shape. The upper surface lift load would have to be taken in tension in the membrane. The structure appears to be the greatest problem.

C. UPPER SURFACE FLUID (FLUID CAMBER)

The concept of using a layer of fluid on the upper surface to change the apparent thickness to the oncoming flow was pursued to some detail early in the project. The following is an approximate analysis for such a scheme.

In order to change the lift coefficient of the foil from .4 to .9, the camber must change from 2.2% of chord to 5.2% of chord, or a change of 3% of chord. In order to change the camber by 3%, the thickness must be changed by 6% of chord.

The proposed scheme of using "fluid camber" to change a foil's lift characteristic calls for pumping fluid from the leading edge on one surface of the foil and retrieving the same fluid back into the foil at the trailing edge.



After some fruitless searching for a potential flow model for the concentrated jet at the leading edge, a half-plane source and sink were chosen as a simple model of both entry and exit of the flow. The stream function for a source and sink of equal strength m and separated by a distance d is:

$$\psi = \frac{m}{2\pi} \operatorname{Tan}^{-1} \frac{-2y(1/2)d}{x^2+y^2-(1/2 \cdot d)^2} \qquad (1)$$

The stream function for a uniform flow in the x direction at speed U_0 is:

$$\psi = U_0 y \tag{2}$$

For the superposition of these flows:

$$\psi = U_0 y + \frac{m}{2\pi}\mathrm{Tan}^{-1} \frac{-2y(1/2)d}{x^2+y^2-((1/2)d)^2} \tag{3}$$

We need the half plane thickness of the ellipse formed at $\psi = 0$ to be 6% of chord, or $y/d = .06$. With $\psi = 0$, $x = 0$, $y = .06d$, we can solve for m.

$$\frac{m}{U_0} = \frac{-(.06d)2\pi}{\mathrm{Tan}^{-1}(\mathrm{Tan}\ B)} \tag{4}$$

where

$$\mathrm{Tan}(B) = \frac{-2(.06)(1/2)d}{(.06d)^2-((1/2)d)^2} \tag{5}$$

$$\mathrm{Tan}(B) = .243 \tag{6}$$

$$B = -166.67° \frac{\pi}{180} \tag{7}$$

$$m = .1295 \cdot d \cdot U_0 \tag{8}$$

But we need only 1/2 this flow but across the span of the foil.

$$\frac{ms}{2} = .0648 \cdot U_0 \cdot ds \tag{9}$$

$$Q = .0648 \cdot U_0 \cdot A \ \mathrm{ft}^3/\mathrm{sec} \tag{10}$$

For an ocean hydrofoil ship of 300 tons at 40 knots with lift coefficient of .4, requires this area:

$$A = \frac{L}{(1/2)\rho V^2 C_l} \tag{11}$$

$$A = \frac{300 \cdot 2240}{(1/2)(2)(67.5)^2(.4)} = 368 \text{ ft}^2 \tag{12}$$

This gives us a required flow rate for the "fluid camber" system of:

$$Q = .0648\ (67.5)(368) = 1610 \text{ ft}^3/\text{sec}$$
$$\text{or} = 7.22 \times 10^5 \text{ gal/min} \tag{13}$$

The "fluid camber" scheme is not feasible for the full scale hydrofoil ship.

III. CAMBER AND ANGLE OF ATTACK CHANGERS

A. ROTATING FOIL/PARTIAL CHORD FLUID (JET TAB)

This scheme would utilize the jet momentum and effect on the overall flow to force the Alpha motion. Reference (7) discusses such a scheme. Hopefully, the flow rate required in the jet could be reduced from that in a jet flap system.

B. TRANSLATING FOIL/PARTIAL CHORD MECHANICAL (SUSPENDED FLAP)

C. TRANSLATING FOIL/PARTIAL CHORD FLUID (SUSPENDED JET FLAP)

Both schemes would use relative motion of a part of the chord to reduce the excursion magnitude of the foil. These schemes seem worth further analysis.

IV. LIFT SPOILERS

A. MECHANICAL (SPOILERS)

Mechanical Spoilers have been used in aircraft as ailerons. The scheme for a hydrofoil ship wave alleviation would require the foil to be at the angle of attack necessary to produce the maximum lift needed. Spoilers would then reduce that lift down to the minimum lift needed. The power and weight for such a system will be small, but the drag is expected to be great.

B. FLUID (VENTILATION)

The scheme of ventilating portions of the lift distribution has been developed by SUPRAMAR. However, the same problems of drag enter as in Section A. It may be that overall system power for such a scheme would be small compared to full incidence.

APPENDIX B

DERIVATION OF $\overline{MOMTBW}$
FOR SECTION IV-B

FROM 2-D LINEAR FLAT PLATE ANALYSIS:

$$\Delta L_w = \frac{\partial C_\ell}{\partial \alpha} \Delta \alpha \frac{\rho}{2} U^2 c$$

but $$\frac{\partial C_\ell}{\partial \alpha} = 2\pi$$

and $$\Delta \alpha = \frac{|\dot{y}|}{U} \quad , \quad c = 2b$$

$$\therefore \quad \Delta L_w = 2\pi \rho U b |\dot{y}|$$

FOR UNSTEADY ANALYSIS

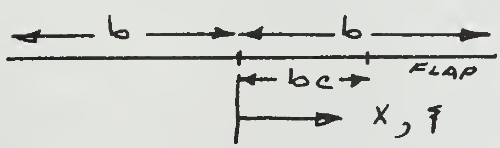
$$\Delta L_w = 2\pi \rho U b |\dot{y}| \cdot |\overline{C(K)}|$$

AND $$\underset{\text{about } ba}{\Delta M_w} = b(\tfrac{1}{2} + a) \Delta L_w$$

$$\therefore \quad \text{THE } \underset{\text{ABOUT } bc}{\overline{\Delta M_w}} = f(\underset{\text{FLAP}}{l_w}, b - bc)$$

AND $$\underset{\text{FLAP}}{\overline{l_w}} = \int_{\text{FLAP}} d l_w \cdot \overline{C(K)}$$

$$\overline{\Delta M_w} = \int_{\text{FLAP}} (b - b\xi) \, d l_w(\xi) \cdot \overline{C(K)}$$



$\xi = 1, \; x = +b$

$$\frac{d\,l_\omega(\xi)}{2\rho U b |\eta|} = \sqrt{\frac{1+\xi}{1-\xi}}\; d\xi$$

SO

$$\frac{\overline{\Delta m_w}}{2\rho U b^2 \eta} = \int_c^{+1} \sqrt{\frac{1+\xi}{1-\xi}}\,(c-\xi)\, d\xi \cdot \overline{C(K)}$$

THE INTEGRAL CAN BE BROKEN INTO TWO PARTS.

$$I = \int_c^{+1} \sqrt{\frac{1+\xi}{1-\xi}} \cdot c\, d\xi \qquad II = \int_c^{+1} \sqrt{\frac{1+\xi}{1-\xi}}\,(-\xi)\, d\xi$$

FIRST WORKING WITH I.

$$c\int_c^{+1} \sqrt{\frac{1+\xi}{1-\xi}}\, d\xi \;=\; \text{OF THE FORM CRC \#64}$$

CRC #64 IF

$$\sqrt{u} = \sqrt{a+bx}$$
$$\sqrt{v} = \sqrt{a'+b'x}$$
$$K = ab' - a'b$$

THEN:

$$\int_{x_0}^{+1} \frac{\sqrt{v}}{\sqrt{u}}\, dx = \frac{\sqrt{uv}}{b} - \frac{K}{2b}\int_{x_0}^{+1} \frac{dx}{\sqrt{uv}}$$

FOR OUR CASE:

$$\sqrt{u} = \sqrt{1-\xi} \qquad a = 1 \quad a' = 1$$
$$\sqrt{v} = \sqrt{1+\xi} \qquad b = -1 \quad b' = 1$$
$$K = 2$$

$$\therefore\quad c\int_c^{+1} \sqrt{\frac{1+\xi}{1-\xi}}\, d\xi = c\left[\frac{\sqrt{1-\xi^2}}{-1} + \underbrace{\int_c^{+1} \frac{d\xi}{\sqrt{1-\xi^2}}}_{III}\right]$$

WORKING WITH INTEGRAL III

$$\int_{C}^{+1} \frac{d\xi}{\sqrt{1-\xi^2}} = \text{OF THE FORM CRC \# 167} = -\cos^{-1}\xi \Big|_{C}^{+1}$$

SO INTEGRAL I BECOMES:

$$C\int_{C}^{+1} \sqrt{\frac{1+\xi}{1-\xi}}\, d\xi = \boxed{C\left[\frac{\sqrt{1-\xi^2}}{-1} - \cos^{-1}\xi\right]_{C}^{+1}}$$

WORKING WITH II

$$\int_{C}^{+1} \sqrt{\frac{1+\xi}{1-\xi}}\,(-\xi)\, d\xi = \text{OF THE FORM CRC \#6}$$

$$\text{CRC \#6} \quad \int_{x_0}^{+1} \frac{dV}{dx}\, u\, dx = UV - \int_{x_0}^{+1} V \frac{du}{dx}\, dx$$

$$\text{IF} \quad \frac{dV}{d\xi} = \sqrt{\frac{1+\xi}{1-\xi}} \quad \text{THEN FROM CRC \# 64}$$

$$V = \frac{\sqrt{1-\xi^2}}{-1} - \cos^{-1}\xi$$

SO

$$\int_{C}^{+1} \sqrt{\frac{1+\xi}{1-\xi}}\,(-\xi)\, d\xi = -\xi\left(\frac{\sqrt{1-\xi^2}}{-1} - \cos^{-1}\xi\right) + \underbrace{\int_{C}^{+1}\left(\frac{\sqrt{1-\xi^2}}{-1} - \cos^{-1}\xi\right) d\xi}_{\text{IV}}$$

WORKING WITH INTEGRAL IV

$$\int_c^{+1}\left(\frac{\sqrt{1-\xi^2}}{-1}-\cos^{-1}\xi\right)d\xi = \underbrace{\int_c^{+1}\left(\frac{\sqrt{1-\xi^2}}{-1}\right)d\xi}_{\mathrm{V}} + \underbrace{\int_c^{+1}\left(-\cos^{-1}\xi\right)d\xi}_{\mathrm{VI}}$$

WORKING WITH INTEGRAL V

$$\int_c^{+1}\left(\frac{\sqrt{1-\xi^2}}{-1}\right)d\xi = \text{OF THE FORM CRC \#166}$$

$$\therefore \int_c^{+1}\left(\frac{\sqrt{1-\xi^2}}{-1}\right)d\xi = -\tfrac{1}{2}\left(\xi\sqrt{1-\xi^2}+\sin^{-1}\xi\right)\Big|_c^{+1}$$

WORKING WITH INTEGRAL VI

$$\int_c^{+1}\left(-\cos^{-1}\xi\right)d\xi = \text{OF THE FORM CRC \#339}$$

$$\therefore \int_c^{+1}\left(-\cos^{-1}\xi\right)d\xi = -\xi\cos^{-1}\xi+\sqrt{1-\xi^2}\Big|_c^{+1}$$

SO INTEGRAL II BECOMES

$$\int_c^{+1}\sqrt{\frac{1+\xi}{1-\xi}}(-\xi)d\xi = \boxed{\begin{aligned}&\xi\sqrt{1-\xi^2}+\xi\cos^{-1}\xi-\tfrac{1}{2}\left(\xi\sqrt{1-\xi^2}+\sin^{-1}\xi\right)\\&-\xi\cos^{-1}\xi+\sqrt{1-\xi^2}\quad\Big|_c^{+1}\end{aligned}}$$

INTEGRALS I & II EVALUATED:

$$\boxed{\left(-1+\tfrac{1}{2}c\right)\sqrt{1-c^2}+\left(c-\tfrac{1}{2}\right)\cos^{-1}c}$$

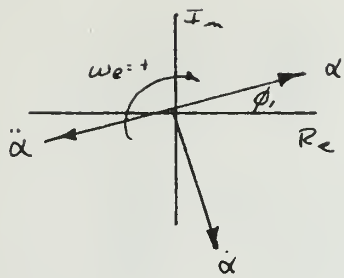
$\therefore$ $$\frac{\overline{\Delta M_\omega}}{2\rho V b^2 \dot{\eta}} = \left[\left(-1+\tfrac{1}{2}c\right)\sqrt{1-c^2} + \left(c-\tfrac{1}{2}\right)\cos^{-1}c\right]\overline{C(k)}$$

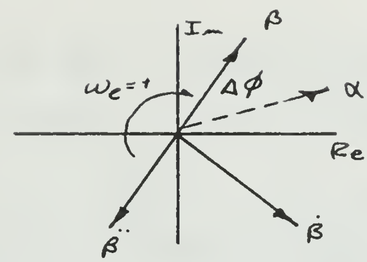
ABOUT bc

OR $$\overline{\Delta M_\omega} = \overline{MOMTBW} = \left[\left(-1+\tfrac{1}{2}c\right)\sqrt{1-c^2} + \left(c-\tfrac{1}{2}\right)\cos^{-1}c\right] \times \overline{LIFTW}/\pi$$

APPENDIX C

DETAILS OF THE IMPLICIT SOLUTION FOR ALPHA AND BETA FOR SECTION IV-D





$$
\begin{array}{lll}
 & Re & Im \\
\alpha = & \alpha_0 \cos\phi_1 & \alpha_0 \sin\phi_1 \\
\dot{\alpha} = & \alpha_0 w_e \sin\phi_1 & \alpha_0 w_e(-\cos\phi_1) \\
\ddot{\alpha} = & \alpha_0 w_e^2(-\cos\phi_1) & \alpha_0 w_e^2(-\sin\phi_1)
\end{array}
$$

$$
\begin{array}{lll}
\beta & = \beta_0/\alpha_0\, \alpha_0 \cos(\phi_1 + \Delta\phi) & \beta_0/\alpha_0\, \alpha_0 \sin(\phi_1 + \Delta\phi) \\
\dot{\beta} & = \beta_0/\alpha_0\, \alpha_0 w_e(\sin\phi_1 + \Delta\phi) & \beta_0/\alpha_0\, \alpha_0 w_e(-\cos\phi_1 + \Delta\phi) \\
\ddot{\beta} & = \beta_0/\alpha_0\, \alpha_0 w_e^2(-\cos\phi_1 + \Delta\phi) & \beta_0/\alpha_0\, \alpha_0 w_e^2(-\sin\phi_1 + \Delta\phi)
\end{array}
$$

$$
-L_w = \pi\rho b^2 \left[\begin{array}{ll} U\alpha_0 w_e \sin\phi_1 & , \; U\alpha_0 w_e(-\cos\phi_1) \\ -ba\,\alpha_0 w_e^2(-\cos\phi_1) & , \; -ba\,\alpha_0 w_e^2(-\sin\phi_1) \end{array}\right]
$$

$$
-\rho b^2 \left[\begin{array}{ll} U c_1\, \beta_0/\alpha_0\, \alpha_0 w_e \sin(\phi_1 + \Delta\phi) & , \; U c_1\, \beta_0/\alpha_0\, \alpha_0 w_e(-\cos\phi_1 + \Delta\phi) \\ + b c_2\, \beta_0/\alpha_0\, \alpha_0 w_e^2(-\cos\phi_1 + \Delta\phi) & , \; b c_2\, \beta_0/\alpha_0\, \alpha_0 w_e^2(-\sin\phi_1 + \Delta\phi) \end{array}\right]
$$

$$
+2\pi\rho U b \left[\begin{array}{ll} U\alpha_0 \cos\phi_1 & , \; U\alpha_0 \sin\phi_1 \\ + b(\tfrac{1}{2} - a)\alpha_0 w_e \sin\phi_1 & , \; b(\tfrac{1}{2} - a)\alpha_0 w_e(-\cos\phi_1) \end{array}\right]
\times \left[\, Re\, \overline{C(k)} \;,\; Im\, \overline{C(k)} \,\right]
$$

$$
+2\pi\rho U b \left[\begin{array}{ll} \dfrac{U c_3}{\pi} \dfrac{\beta_0}{\alpha_0} \alpha_0 \cos(\phi_1 + \Delta\phi) & , \; \dfrac{U c_3}{\pi} \dfrac{\beta_0}{\alpha_0} \alpha_0 \sin(\phi_1 + \Delta\phi) \\ + \dfrac{b c_4}{2\pi} \beta_0/\alpha_0\, \alpha_0 w_e \sin(\phi_1 + \Delta\phi) & , \; \dfrac{b c_4}{2\pi} \beta_0/\alpha_0\, \alpha_0 w_e(-\cos\phi_1 + \Delta\phi) \end{array}\right]
\times \left[\, Re\, \overline{C(k)} \;,\; Im\, \overline{C(k)} \,\right]
$$

but $\quad \sin(\phi_1 + \Delta\phi) = \sin\phi_1 \cos\Delta\phi + \cos\phi_1 \sin\Delta\phi$

$\qquad \cos(\phi_1 + \Delta\phi) = \cos\phi_1 \cos\Delta\phi - \sin\phi_1 \sin\Delta\phi$

$$(a + ib)(c + id) = (ac - bd) + i(bc + ad)$$

let $\quad S_1 = \sin\phi_1 \quad S_\Delta = \sin\Delta\phi \qquad F = Re\,\overline{C(k)}$

so $\quad C_1 = \cos\phi_1 \quad C_\Delta = \cos\Delta\phi \qquad G = Im\,\overline{C(k)}$

$$-L_w = \pi\rho b^2 \begin{bmatrix} U\alpha_0 w_e S_1 & , U\alpha_0 w_e(-C_1) \\ -ba\,\alpha_0 w_e^2(-C_1) & , -ba\,\alpha_0 w_e^2(-S_1) \end{bmatrix}$$

$$-\rho b^2 \begin{bmatrix} U c_1 \beta_0/\alpha_0\, \alpha_0 w_e (S_1 C_\Delta + C_1 S_\Delta) & , U c_1 \beta_0/\alpha_0\, \alpha_0 w_e(-C_1 C_\Delta + S_1 S_\Delta) \\ + b c_2 \beta_0/\alpha_0\, \alpha_0 w_e^2(-C_1 C_\Delta + S_1 S_\Delta) & , b c_2 \beta_0/\alpha_0\, \alpha_0 w_e^2 (-S_1 C_\Delta - C_1 S_\Delta) \end{bmatrix}$$

$$2\pi\rho U b \begin{bmatrix} U\alpha_0 (C_1 F - S_1 G) & , U\alpha_0 (C_1 G + S_1 F) \\ + b(1/2 - a)\alpha_0 w_e (S_1 F - (-C_1 G)) & , b(1/2 - a)\alpha_0 w_e (S_1 G - C_1 F) \end{bmatrix}$$

$$2\pi\rho U b \left[\begin{array}{l} \frac{U c_3}{\pi} \beta_0/\alpha_0\, \alpha_0 (C_1 C_\Delta F - S_1 S_\Delta F - S_1 C_\Delta G - C_1 S_\Delta G), \\ \qquad\qquad \frac{U c_3}{\pi} \beta_0/\alpha_0\, \alpha_0 (C_1 C_\Delta G - S_1 S_\Delta G + S_1 C_\Delta F + C_1 S_\Delta F) \\ + \frac{b c_4}{2\pi} \beta_0/\alpha_0\, \alpha_0 w_e (S_1 C_\Delta F + C_1 S_\Delta F + C_1 C_\Delta G - S_1 S_\Delta G), \\ \qquad\qquad + \frac{b c_4}{2\pi} \beta_0/\alpha_0\, \alpha_0 w_e (S_1 C_\Delta G + C_1 S_\Delta G - C_1 C_\Delta F + S_1 S_\Delta F) \end{array}\right]$$

DIVIDING THROUGH BY $C_1 = \cos\phi_1$, α_0, ρ, b

LET $\qquad T = S_1/C_1$

AND SEPARATING REAL & IMAGINARY PARTS

LEAVES:

$$\frac{-Re\,\overline{L_\omega}}{C_1 \alpha_0 \rho b} = T \overset{Y_1}{\begin{bmatrix} \pi\, w_e\, v \\ - b\, \beta_0/\alpha_0\, w_e\, v\, c_1\, C_\Delta \\ - b\, \beta_0/\alpha_0\, w_e\, b\, c_2\, w_e\, S_\Delta \\ - 2\pi\, v\, v\, G \\ + 2\pi\, v\, b(1/2 - a)\, w_e\, F \\ - 2 v\, \beta_0/\alpha_0\, v\, c_3\, F\, S_\Delta \\ - 2 v\, \beta_0/\alpha_0\, v\, c_3\, G\, C_\Delta \\ + 2 v\, \beta_0/\alpha_0\, \frac{b\, c_4}{2}\, w_e\, F\, C_\Delta \\ - 2 v\, \beta_0/\alpha_0\, \frac{b\, c_4}{2}\, w_e\, G\, S_\Delta \end{bmatrix}}$$

$$+ \overset{Y_2}{\begin{bmatrix} \pi\, b\, w_e\, b a\, w_e \\ - b\, \beta_0/\alpha_0\, w_e\, v\, c_1\, S_\Delta \\ + b\, \beta_0/\alpha_0\, w_e\, b\, c_2\, w_e\, C_\Delta \\ + 2\pi\, v\, v\, F \\ + 2\pi\, v\, b(1/2 - a)\, w_e\, G \\ + 2 v\, \beta_0/\alpha_0\, v\, c_3\, F\, C_\Delta \\ - 2 v\, \beta_0/\alpha_0\, v\, c_3\, G\, S_\Delta \\ + 2 v\, \beta_0/\alpha_0\, \frac{b\, c_4}{2}\, w_e\, F\, S_\Delta \\ + 2 v\, \beta_0/\alpha_0\, \frac{b\, c_4}{2}\, w_e\, G\, C_\Delta \end{bmatrix}}$$

$$-\frac{\overline{I_m L_\omega}}{C_1 \alpha_0 b \ell} = T \begin{array}{c} X_1 \\ \left[\begin{array}{l} \pi b \omega_e b a \omega_e \\ - b \beta_0/\alpha_0 \omega_e \upsilon c_1 S_\Delta \\ + b \beta_0/\alpha_0 \omega_e b c_2 \omega_e C_\Delta \\ 2\pi \upsilon \upsilon F \\ 2\pi \upsilon b (1/2 - a) \omega_e G \\ - 2 \upsilon \beta_0/\alpha_0 \upsilon c_3 G S_\Delta \\ + 2 \upsilon \beta_0/\alpha_0 \upsilon c_3 F C_\Delta \\ + 2 \upsilon \beta_0/\alpha_0 \frac{b c_4}{2} \omega_e G C_\Delta \\ + 2 \upsilon \beta_0/\alpha_0 \frac{b c_4}{2} \omega_e F S_\Delta \end{array}\right] \end{array}$$

$$+ \begin{array}{c} X_2 \\ \left[\begin{array}{l} - \pi b \omega_e \upsilon \\ + b \beta_0/\alpha_0 \omega_e \upsilon c_1 C_\Delta \\ + b \beta_0/\alpha_0 \omega_e b \omega_e c_2 S_\Delta \\ + 2\pi \upsilon \upsilon G \\ - 2\pi \upsilon b (1/2 - a) \omega_e F \\ + 2\pi \beta_0/\alpha_0 \upsilon G c_3 C_\Delta \\ + 2 \upsilon \beta_0/\alpha_0 \upsilon F c_3 S_\Delta \\ + 2 \upsilon \beta_0/\alpha_0 \frac{b c_4}{2} \omega_e G S_\Delta \\ - 2 \upsilon \beta_0/\alpha_0 \frac{b c_4}{2} \omega_e F C_\Delta \end{array}\right] \end{array}$$

WHERE

$$C_1 = c\sqrt{1-c^2} - \cos^{-1}c$$

$$C_2 = c\cos^{-1}c - 1/3\,(2 + c^2)$$

$$C_3 = \sqrt{1-c^2} + \cos^{-1}c$$

$$C_4 = (1 - 2c)\cos^{-1}c + (2-c)\sqrt{1-c^2}$$

AND

$$C_5 = 1/3\sqrt{1-c^2}\,(c^2 - 1) - (c-a)\cdot C_1$$

$$C_6 = (1/8 + c^2)\cos^{-1}c - 1/8\,c\sqrt{1-c^2}\,(7 + 2c^2) + (c-a)\cdot(-C_2)$$

$$C_7 = a\,C_1 + 1/3\left(\sqrt{1-c^2}\right)^3 + C_2$$

$$C_8 = 2c\sqrt{1-c^2}\cos^{-1}c - (1-c^2) - (\cos^{-1}c)^2$$

$$C_9 = 1/4\,c\sqrt{1-c^2}\cos^{-1}c\,(7 + 2c^2) - (1/8 + c^2)(\cos^{-1}c)^2 - 1/8\,(1-c^2)(1/2\,c^2 + 4)$$

$$C_{10} = (1 + 1/2\,c)\sqrt{1-c^2} - (c + 1/2)\cos^{-1}c$$

$$C_{11} = (1 - 1/2\,c)\sqrt{1-c^2} + (1/2 - c)\cos^{-1}c$$

APPENDIX D

PROGRAM LISTINGS FOR SECTION IV-H

I. COMPUTER PROGRAM FOILDYN (WRITING)

II. COMPUTER PROGRAM FOILDYN (PLOTTING)

```
C*********  PROGRAM FOILDYN     WRITING       MIKE TERRY
      COMPLEX COM1,COM2,COM3,COM4,COM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1     ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
     2     MOMTBB,LIFTOT,MOMTOT,MOMTBT,POWERA,POWERB,MOMAPA,MOMAPB,LIF
      COMMON  RO,B,A,C,MODE,BA,DPHE,PI,G,HW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1     C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2     BDOT,BDDOT,C11
C**********READ IN NUMBER OF FOIL RUNS
      READ (5,102)  MMAX
      M=1
C********READ IN FOIL INFORMATION  RO = SLUGS PER FT3, B = 1/2 CHORD IN
C********FT, A = LIFT POINT FROM MIDCHORD AFT IN RATIO OF B, C = FLAP
C********POINT FROM MIDCHORD AFT  IN RATIO OF B, MODE = 1 FOR ALPHA ONLY
C********2 FOR BETA ONLY, 3 FOR ALPHA& BETA, BA = RATIO OFMAGBETA TO MAG
C********ALPHA, DPHE = ANGLE BETWEEN BETA & ALPHA IN RADIANS
C**********FIA =FOIL INERTIA IN SLUGS . FT2/FT SPAN
C**********FIB=CONTROL SURFACE INERTIA IN SLUGS FT2/FT SPAN
 5050 WRITE (6,118)  M
  118 FORMAT (1H1,14HFOIL NUMBER = ,1I2)
      READ (5,101) RO,B,A,C,MODE,BA,DPHE
  101 FORMAT(4F10.5,1I2,2F10.5)
      READ (5,113)  FIA,FIB
  113 FORMAT(2F20.5)
C********SET UP FOR MULTIPLE WAVE RUNS NMAX
      READ (5,102) NMAX
  102 FORMAT (1I2)
      N = 1
      PI = 3.1416
      G = 32.2
C********IF C = 0 OR NOT CALCULATE CONSTANTS C1 TO C10
      IF(C.EQ.0.0) GO TO 200
      CCC = 1.0 - C**2
      SRTC = SQRT(CCC)
      ACC = ATAN2(SRTC,C)
      GO TO 300
  200 CCC = 1.0
      SRTC = 1.0
```

```
      ACC = PI/2.C
  3C0 C1 = C*SRTC - ACC
      C2 = C*ACC - .332*(2.0 + C**2)
      C3 =    SRTC + ACC
      C4 = (1.C - 2.0*C)*ACC + (2.0 - C)*SRTC
      C5 = .332*SRTC*(C**2 - 1.0)  -  (C - A)*C1
      C6 = (C.125 + C**2)*ACC - 0.125*C*SRTC*(7.0 + 2.0*C**2) + (C - A)
     1     *(-C2)
      C7 = A*C1 + 0.332*SRTC**3 + C2
      C8 = 2.C*C*SRTC*ACC - CCC - ACC**2
      C9 = 0.25*C*SRTC*ACC*(7.0 + 2.0*C**2) - (C.125 + C**2)*ACC**2
     1     -C.125*CCC*(5.0*C**2 + 4.0)
      C1C = (1.C + 0.5*C)*SRTC - (C + 0.5)*ACC
      C11 = (1.0  -  0.5*C)*SRTC    +    (C.5  -  C)*ACC
      WRITE (6,301) C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11
  301 FORMAT (11F10.5)
C********READ IN WAVE INFORMATION HW = WAVE HALF HEIGHT IN FT, WL = WAVE
C********LENGTH IN FT, U = SHIP VELOCITY IN FT/SEC, SI = ANGLE BETWEEN U
C********AND ONCOMING WAVE PHASE VELOCITY IN RADIANS, PHEW = REF WAVE
C*********ORBITAL UPWASH VELOCITY PHASE ANGLE IN RADIANS
 400  READ (5,1C3) HW,WL,U,SI,PHEW
  103 FORMAT (5F10.5)
      WRITE (6,1C8)  N
  108 FORMAT (1H1,13HRUN NUMBER = ,I12,9HFOIL DATA)
      WRITE (6,109)  RC,B,A,C,MODE,BA,CPHE,FIA,FIB
  109 FORMAT (1HC,10HDENSITY = ,1F10.5,12HSEMICHORD = ,F10.5,
     1  13HLIFT POINT = ,F10.5,13HFLAP POINT = ,F1C.5,/,1HC,
     2     14HCONTRL MODE = ,I12,16HBETA TO ALPHA = ,F10.5,
     3     22HBETA TO ALPHA ANGLE = ,F10.5,15HFOIL INERTIA = ,
     4     F20.5,/,1H0,25HCONTRL SURFACE INERTIA = ,F2C.5)
      WRITE ( 6,110) HW,WL,U,SI,PHEW
  110 FORMAT(1H0,9HWAVE DATA,/,1H0,14HWAVE HEIGHT = ,F10.5,
     1  14HWAVE LENGTH = ,F10.5,16HVEHICLE SPEED = ,F1C.5,/,1HC,
     2  17HWAVE DIRECTION = ,F1C.5,19HWAVE PHASE ANGLE = ,F10.5)
C*********CALCULATE W2=WAVE FREQ **2 IN RAD/SEC
      W2 = (2C2.0)/WL
C*********CALCULATE ENCOUNTER FREQ IN RAD/SEC FROM WL, U, SI
```

```
      WE = SQRT(W2) + (W2*U*COS(SI))/G
      IF(U.EQ.0.0)  GO TO 900
C*********CALCULATE CA2=REDUCED FREQUENCY BASED ON  SEMICHORD
      CA2 = WE*B/U
      CA = ABS(CA2)
      CALL BESJ(CA,0,B0K,0.01,IER)
      CALL BESJ(CA,1,B1K,0.01,IER)
C**********TEST FOR SMALL CA BEFORE TRYING BESY
      IF(CA - 0.05 )  600, 600, 700
  600 CK = (1.0,-0.000005)
      GO TO 800
  900 CK = (0.5,-0.0000025)
      GO TO 800
  700 CALL BESY(CA,1,Y1K,IER)
      CALL BESY(CA,0,Y0K,IER)
      COM1 = CMPLX(B1K,-Y1K)
      COM2 = CMPLX(B0K,-Y0K)
      COM3 = (0.0,1.0)
      COM4 = COM3*COM2
      CK = COM1/(COM1 + COM4)
  800 WRITE(6,104) WE,CK,CA2
  104 FORMAT (1H0,17HENCOUNTER FREQ = ,F10.5,17HTHEODORSEN FUN.= ,
     1       2F10.5,15HREDUCED FREQ = ,F10.5)
C*********CALCULATE LIFT DUE TO WAVES
 1000 CALL WAVLIF(W2,CA,       MOMTW,MOMTBW)
C*********SOLVE FOR ALPHA & BETA
 2000 CALL CONTRL(ALPH ,BET )
      IF(MODE.EQ.1) GO TO 3000
      IF(MODE.EQ.2) GO TO 4000
C*********CALCULATE LIFT DUE TO ALPHA
 3000 CALL LIFA(LIFTA,MOMTA,MOMTBA)
      IF(MODE.EQ.1) GO TO 5000
C*********CALCULATE LIFT DUE TO BETA
 4000 CALL LIFB(LIFTB,MOMTB,MOMTBB)
 5000 IF(MODE.EQ.1) GO TO 7000
      IF(MODE.EQ.2) GO TO 8000
      GO TO 9000
```

```
C*********INITIALIZE ALPHA & BETA
 7000 LIFTB = (0.0,0.0)
      MOMTB = (0.0,0.0)
      MOMTBB = (0.0,0.0)
      GO TO 9000
 8000 LIFTA = (0.0,0.0)
      MOMTA = (0.0,0.0)
      MOMTBA = (0.0,0.0)
C********* TOTALS
 9000 LIFTOT = LIFTW + LIFTA + LIFTB
      MOMTOT = MOMTW + MOMTA + MOMTB
      MOMTBT = MOMTBW + MOMTBA + MOMTBB
      WRITE (6,105) LIFTOT
  105 FORMAT (1H1,13HTOTAL LIFT = ,2F20.10)
      WRITE (6,106)  MOMTOT
  106 FORMAT (1H0,15HTOTAL MOMENT = ,2F20.10)
      WRITE (6,107) MOMTBT
  107 FORMAT (1H0,33HTOTAL MOMENT ON CONTRL SURFACE = ,2F20.10)
      IF(MODE.EQ.1) GO TO 9002
      IF(MODE.EQ.2) GO TO 9003
C**********MOMENT APPLIED EQUALS I*ADDOT MINUS SUM HYD MOMENT
 9002 MOMAPA = FIA*ADDOT  - MOMTOT
      WRITE(6,114)  MOMAPA
  114 FORMAT(1H0,10H**********,/,1H0,26HREQUIRED MOMENT APPLIED = ,
     1       2F20.10)
      POWERA = ADOT*MOMAPA
      WRITE (6,111)  POWERA
  111 FORMAT(1H0,18HPOWER FOR ALPHA = ,2F20.10)
      CALL FTAN(ADOT,ADMAG,PHEAD)
      CALL FTAN(MOMAPA,PMAMAG,PHEMPA)
      WRITE(6,117)  ADMAG,PHEAD,PMAMAG,PHEMPA
  117 FORMAT(1H0,15HRATE MAG-ARG = ,2F10.5,17HMOMENT MAG-ARG = ,2F20.10)
C**********FINDING MAG & ANGLE FOR AVERAGE POWER CALC
      PHEPA = PHEAD - PHEMPA
      PFACTA = COS(PHEPA)
      PAVEA = 0.5*ADMAG*PMAMAG*PFACTA
      WRITE (6,115)  PAVEA,PFACTA
```

```
  115 FORMAT(1H0,35HAVERAGE POWER REQUIRED FOR ALPHA = ,F20.5,
     1      22HDUE TO POWER FACTOR = ,F10.5)
      IF(MODE.EQ.1) GO TO 9001
 9003 MOMAPB = FIB*BDDOT  -  MOMTBT
      WRITE(6,114)  MOMAPB
      POWERB = BDOT*MOMAPB
      WRITE (6,112)  POWERB
  112 FORMAT (1H0,17HPOWER FOR BETA = ,2F20.10)
      CALL FTAN(BDOT,BDMAG,PHEBD)
      CALL FTAN(MOMAPB,PMBMAG,PHEMPB)
      WRITE(6,117)  BDMAG,PHEBD,PMBMAG,PHEMPB
      PHEPB = PHEBD  -  PHEMPB
      PFACTB = COS(PHEPB)
      PAVEB = 0.5*BDMAG*PMBMAG*PFACTB
      WRITE(6,116) PAVEB,PFACTB
  116 FORMAT(1H0,34HAVERAGE POWER REQUIRED FOR BETA = ,F20.5,
     1      22HDUE TO POWER FACTOR = ,F10.5)
 9001 N = N + 1
      IF (N - NMAX)  400, 400, 6000
 6000 M = M + 1
      IF(M  -  MMAX)  5050,5050,5080
 5080 STOP
      END
      SUBROUTINE WAVLIF(W2,CA,        MOMTW,MOMTBW)
      COMPLEX COM1,COM2,COM3,COM4,COM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1      ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
     2      MOMTBB,LIFTOT,MOMTOT,MOMTBT,POWERA,POWERB,MOMAPA,MOMAPB,LIF
      COMMON  RO,B,A,C,MODE,BA,DPHE,PI,G,HW,WL,L,SI,PHEW,C1,C2,C3,C4,
     1      C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2      BDOT,BDDOT,C11
      WRITE(6,100)
  100 FORMAT(1H0,10H**********,/,1H0,19HFORCES DUE TO WAVES)
C*********PEAK VELOCITY  UPWASH
      UW = HW*SQRT(W2)
      CUW = UW*COS(PHEW)
      SUW = UW*SIN(PHEW)
      UPW = CMPLX(CUW,SUW)
```

```
      WRITE(6,800) UPW
  800 FORMAT(1H0,24HWAVE UPWASH REAL-IMAG = ,2F10.5)
      COM3 = (0.0,1.0)
      CALL BESJ(CA,0,B0K,0.01,IER)
      CALL BESJ(CA,1,B1K,0.01,IER)
      COM5 = CMPLX(B0K,-B1K)
      LIF = CK*COM5    +  COM3*B1K
C**********CHK FOR + _WE
      IF(WE - 0.0)  500,500,600
  600 LIFTW = 2.0*PI*RO*U*B*UPW*CONJG(LIF)
      GO TO 700
  500 LIFTW = 2.0*PI*RO*U*B*UPW*LIF
C**********MOMENT ABOUT Y
  700 MOMTW = B*(0.5  +  A)*LIFTW
C**********MOMENT ABOUT C
      MOMTBW =  B*(-C11)*LIFTW/PI
      CALL FTAN(LIFTW,ALIFTW,PHELW)
      CALL FTAN(MOMTW,AMOMW,PHEMW)
      CALL FTAN(MOMTBW,AMOMBW,PHEMBW)
      WRITE(6,200) LIFTW
  200 FORMAT(1H0,22HWAVE LIFT REAL-IMAG = ,2F20.10)
      WRITE(6,300) MOMTW
  300 FORMAT(1H0,24HWAVE MOMENT REAL-IMAG = ,2F20.10)
      WRITE(6,400) MOMTBW
  400 FORMAT(1H0,35HCONTROL SURFACE MOMENT REAL-IMAG = ,2F20.10)
      RETURN
      END
      SUBROUTINE CONTRL(ALPH ,BET )
      COMPLEX COM1,COM2,COM3,COM4,COM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1      ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
     2      MOMTBB,LIFTOT,MOMTOT,MOMTBT,POWERA,POWERB,MOMAPA,MOMAPB,LIF
      COMMON  RO,B,A,C,MODE,BA,DPHE,PI,G,HW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1      C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2      BDOT,BDDOT,C11
C**********CALCULATE CONSTANTS
C********MAKE +DPHE LEADING
      IF(WE - 0.0) 2000,2000,3000
```

```
 3000 XPHE = -DPHE
      GO TO 4000
 2000 XPHE = DPHE
 4000 CDPHE = COS(XPHE)
      SDPHE = SIN(XPHE)
      C1CP = C1*CDPHE
      C2CP = C2*CDPHE
      C3CP = C3*CDPHE
      C4CP = C4*CDPHE
      C1SP = C1*SDPHE
      C2SP = C2*SDPHE
      C3SP = C3*SDPHE
      C4SP = C4*SDPHE
      CKR = REAL(CK)
C**********CHK FOR + _ WE
      IF(WE - 0.0)  900,900,1000
 1000 CKI = -AIMAG(CK)
      GO TO 1100
  900 CKI = AIMAG(CK)
 1100 IF(MODE.EQ.1) GO TO 100
      IF(MODE.EQ.2) GO TO 200
      IF(MODE.EQ.3) GO TO 300
      GO TO 600
  100 WRITE(6,101)
  101 FORMAT(1H1,10H**********,/,1H0,18HMODE IS ALPHA ONLY)
      GO TO 400
  200 WRITE (6,102)
  102 FORMAT(1H1,10H**********,/,1H0,17HMODE IS BETA ONLY)
C**********BETA ONLY EQS
      Y1 = -B*WE*(U*C1CP +B*WE*C2SP) -2.0*U*(U*CKR*C3SP + U*CKI*C3CP -B*
     10.5*WE*CKR*C4CP + B*0.5*WE*CKI*C4SP)
      Y2 = -B*WE*(U*C1SP - B*WE*C2CP)  +2.0*U*(U*CKR*C3CP -U*CKI*C3SP +
     1 B*0.5*WE*CKR*C4SP +B*0.5*WE*CKI*C4CP)
      X1 = -B*WE*(U*C1SP -B*WE*C2CP) +2.0*U*(-U*CKI*C3SP +U*CKR*C3CP +
     1 B*0.5*WE*CKI*C4CP  +B*0.5*WE*CKR*C4SP)
      X2 = -B*WE*(-U*C1CP -B*WE*C2SP)  +2.0*U*(U*CKI*C3CP + U*CKR*C3SP+
     1 B*0.5*WE*CKI*C4SP  -B*0.5*WE*CKR*C4CP)
```

```
      GO TO 500
  300 WRITE (6,103) BA,DPHE
  103 FORMAT(1H1,10H**********,/,1H0,
     1      33HMODE IS ALPHA AND BETA WITH BA = ,F10.5,12H AND DPHE = ,
     2      F10.5)
C**********ALPHA ONLY OR ALPHA + BETA EQS
  400 Y1 = PI*B*WE*U  -B*BA*WE*(U*C1CP  +B*WE*C2SP)  -2.0*PI*U*(U*CKI -
     1 B*(0.5 -A)*WE*CKR)   -2.0*U*BA*(U*CKR*C3SP + U*CKI*C3CP -
     2 B*0.5*WE*CKR*C4CP + B*0.5*WE*CKI*C4SP)
      Y2 = PI*B*WE*B*A*WE  -B*BA*WE*(U*C1SP  -B*WE*C2CP)  +2.0*PI*U*(U*
     1CKR +B*WE*CKI*(0.5 -A)) +2.0*U*BA*(U*CKR*C3CP -U*CKI*C3SP +
     2B*0.5*WE*CKR*C4SP +B*0.5*WE*CKI*C4CP)
      X1 = PI*B**2*WE**2*A  -B*BA*WE*(U*C1SP  -B*WE*C2CP) +2.0*PI*U*(U*
     1CKR + B*(0.5 -A)*WE*CKI) +2.0*U*BA*(-U*CKI*C3SP +U*CKR*C3CP +
     2B*0.5*WE*CKI*C4CP  +B*0.5*WE*CKR*C4SP)
      X2 = -PI*B*WE*U  -B*BA*WE*(-U*C1CP  -B*WE*C2SP)  +2.0*PI*U*(U*
     1CKI  -B*(0.5 -A)*WE*CKR)  +2.0*U*BA*(U*CKI*C3CP +U*CKR*C3SP +
     2  B*0.5*WE*CKI*C4SP  -B*0.5*WE*CKR*C4CP)
  500 ALR = REAL(LIFTW)
      ALI = AIMAG(LIFTW)
      R = ALR/ALI
      Y = Y2  -  R*X2
      X = R*X1  -  Y1
C**********SOLVE FOR ANGLE
      PHEA = ATAN2(Y,X)
      CPHEA = COS(PHEA)
      AAA = Y1*Y/X  +  Y2
C**********SOLVE FOR MAGNITUDE
      ALPH = -ALR/(CPHEA*RC*B*AAA)
      ALPHA = CMPLX(ALPH*COS(PHEA),ALPH*SIN(PHEA))
      BET = BA*ALPH
      PHEB = PHEA + XPHE
      BETA = CMPLX(BET*COS(PHEB),BET*SIN(PHEB))
      IF(MODE.EQ.1) GO TO 700
      IF(MODE.EQ.2) GO TO 800
  700 WRITE(6,104) ALPHA
  104 FORMAT(1H0,18HALPHA REAL-IMAG = ,2F10.5)
```

```
      COM3 = (0.0,1.0)
C**********SOLVE FOR ALPHA DOT AND ALPHA DOUBLE DOT
      ADOT = -COM3*WE*ALPHA
      ADDOT = -WE**2*ALPHA
      WRITE (6,106)  ADOT,ADDOT
  106 FORMAT(1H0,13HALPHA RATE = ,2F10.5,12HALPHA ACC.= ,2F10.5)
      IF(MODE.EQ.1) GO TO 600
  800 WRITE(6,105) BETA
  105 FORMAT(1H0,17HBETA REAL-IMAG = ,2F10.5)
      COM3 = (0.0,1.0)
C**********SOLVE FOR BETA DOT AND BETA DOUBLE DOT
      BDOT = -COM3*WE*BETA
      BDDOT = -WE**2*BETA
      WRITE(6,107)  BDOT,BDDOT
  107 FORMAT(1H0,12HBETA RATE = ,2F10.5,11HBETA ACC.= ,2F10.5)
  600 CONTINUE
      RETURN
      END
      SUBROUTINE LIFA(LIFTA,MOMTA,MOMTBA)
      COMPLEX COM1,COM2,COM3,COM4,COM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1     ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
     2     MOMTBB,LIFTOT,MOMTOT,MOMTBT,POWERA,POWERB,MOMAFA,MOMAPB,LIF
      COMMON  RC,B,A,C,MODE,BA,DPHE,PI,G,HW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1     C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2     BDOT,BDDOT,C11
      WRITE(6,100)
  100 FORMAT(1H0,10H**********,/,1H0,22HFORCES DUE TO ALPHA = )
C**********TEST FOR + _ WE
      IF(WE - 0.0) 500,500,600
  600 LIF = CONJG(CK)
      GO TO 700
  500 LIF = CK
  700 LIFTA = PI*RO*B**2*(U*ADOT -B*A*ADDOT)  +  2.0*PI*RO*U*B*
     1     (U*ALPHA  +  B*(0.5 - A)*ADOT)*LIF
C********** MOMENT ABOUT Y
      MOMTA = PI*RO*B**2*(U**2*ALPHA  - B**2*(0.125 + A**2)*ADDOT)
     1     -2.0*PI*RO*U*B**2*(U*ALPHA + B*(0.5 - A)*ADOT)*(0.5
     2     -(A + 0.5)*LIF)
```

```
C**********MOMENT ABOUT C
      MOMTBA =-RO*B**2*U**2*C1*ALPHA +RO*B**3*U*C7*ADOT-RO*B**4*C6*ADDOT
     1      -2.0*RO*U*B**2*(U*ALPHA + B*(0.5 -A)*ADOT)*(-0.5*C1 + C10*
     2      LIF)
      WRITE (6,200) LIFTA
  200 FORMAT (1H0,30HLIFT DUE TO ALPHA REAL-IMAG = ,2F20.10)
      WRITE (6,300)  MOMTA
  300 FORMAT (1H0,32HMOMENT DUE TO ALPHA REAL-IMAG = ,2F20.10)
      WRITE (6,400)  MOMTBA
  400 FORMAT (1H0,50HMOMENT ON CONTRL SURFACE DUE TO ALPHA REAL-IMAG = ,
     1      2F20.10)
      RETURN
      END
      SUBROUTINE LIFB(LIFTB,MOMTB,MOMTBB)
      COMPLEX COM1,COM2,COM3,COM4,COM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1      ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
     2      MOMTBB,LIFTCT,MOMTCT,MOMTBT,POWERA,POWERB,MOMAPA,MOMAPB,LIF
      COMMON  RO,B,A,C,MODE,BA,OPHE,PI,G,FW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1      C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2      BDOT,BDDOT,C11
      WRITE(6,100)
  100 FORMAT(1H0,10H**********,/,1H0,21HFORCES DUE TO BETA = )
C**********TEST FOR + _ WE
      IF(WE - 0.0) 500,500,600
  600 LIF = CONJG(CK)
      GO TO 700
  500 LIF = CK
  700 LIFTB = -RO*B**2*U*C1*BDOT-RO*B**3*C2*BDDOT + RO*2.0*U*B*
     1      (U*C3*BETA + 0.5*B*C4*BDOT)*LIF
C**********MOMENT ABOUT Y
      MOMTB = -RO*B**2*U**2*C1*BETA -RO*B**3*U*C5*BDOT -RO*B**4*C6*BDDOT
     1      -2.0*RO*U*B**2*(U*C3*BETA + 0.5*B*C4*BDOT)*(0.5 -(A+0.5)*LIF)
C**********MOMENT ABOUT C
      MOMTBB = -RO*B**2*U**2*C8*BETA/PI +RO*B**4*C9*BDDOT/PI
     1      -2.0*RO*U*B**2*(U*C3*BETA/PI +0.5*B*C4*BDOT/PI)*(-0.5*C1 +
     2      C10*LIF)
      WRITE (6,200)  LIFTB
```

```
  200 FORMAT (1H0,29HLIFT DUE TO BETA REAL-IMAG = ,2F20.10)
      WRITE (6,300)  MOMTB
  300 FORMAT (1H0,31HMOMENT DUE TO BETA REAL-IMAG = ,2F20.10)
      WRITE (6,400) MOMTBB
  400 FORMAT (1H0,49HMOMENT ON CONTRL SURFACE DUE TO BETA REAL-IMAG = ,
     1      2F20.10)
      RETURN
      END
      SUBROUTINE FTAN(ARG,AMAG,PHE)
      COMPLEX CCM1,CCM2,CCM3,CCM4,CCM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1      ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
     2      MOMTBB,LIFTCT,MOMTCT,MOMTBT,POWERA,POWERB,MOMAPA,MOMAPB,ARG,
     3      LIF
      COMMON  RO,B,A,C,MODE,BA,DPHE,PI,G,HW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1      C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2      BDOT,BDDOT,C11
C**********FINDS MAG AND ANGLE OF ARG
      X = REAL(ARG)
      Y = AIMAG(ARG)
      IF(X.EQ.0.0) GO TO 200
      PHE = ATAN2(Y,X)
      GO TO 300
  200 IF(Y - 0.0) 400,500,600
  400 PHE = -PI/2.0
      GO TO 300
  500 PHE = PI/4.0
      GO TO 300
  600 PHE = PI/2.0
  300 AMAG = CABS(ARG)
      RETURN
      END
```

```
RUN NUMBER = 1 FOIL DATA

DENSITY= 2.00 SEMICHORD= 4.29000 LIFT POINT= 0.0 FLAP POINT= 0.50000

CONTRL MODE= 3 BETA TO ALPHA= 2.0 BETA TO ALPHA ANGLE= -0.39270 FOIL INERTIA= 140.00

CONTRL SURFACE INERTIA= 8.78000

WAVE DATA

WAVE HEIGHT= 7.00000 WAVE LENGTH= 280.00000 VEHICLE SPEED= 80.00000

WAVE DIRECTION=0.0 WAVE PHASE ANGLE= -1.57080

ENCOUNTER FREQ= 2.64174 THEODORSEN FUN.= 0.78157 -0.18519 REDUCED FREQ= 0.14166

**********

FORCES DUE TO WAVES

WAVE UPWASH REAL-IMAG= -0.00002  -5.94559

WAVE LIFT REAL-IMAG= 4328.92578  -19605.19921

WAVE MOMENT REAL-IMAG= 9285.54296  -42053.14062

CONTROL SURFACE MOMENT REAL-IMAG= -3839.53222  17388.79296
```

```
**********

MODE IS ALPHA AND BETA WITH BA= 2.0 AND DPHE= -0.39270

ALPHA REAL-IMAG= 0.00400  0.03358

ALPHA RATE= 0.08870  -0.01058 ALPHA ACC.= -0.02794  -0.23432

BETA REAL-IMAG= -0.01830  0.06511

BETA RATE= 0.17199  0.04835 BETA ACC.= 0.12772  -0.45436

**********

FORCES DUE TO ALPHA=

LIFT DUE TO ALPHA REAL-IMAG= 414.29028  9287.671875

MOMENT DUE TO ALPHA REAL-IMAG= -2624.10864  20404.14843

MOMENT ON CONTRL SURFACE DUE TO ALPHA REAL-IMAG= -331.32934  -395.16406

**********

FORCES DUE TO BETA=

LIFT DUE TO BETA REAL-IMAG= -4743.16406  10317.41406

MOMENT DUE TO BETA REAL-IMAG= -6869.02734  1632.10546

MOMENT ON CONTRL SURFACE DUE TO BETA REAL-IMAG= 289.69531  -1715.66796
```

```
TOTAL LIFT= 0.05078  -0.11328

TOTAL MOMENT= -207.59375  -20016.88671

TOTAL MOMENT ON CONTRL SURFACE= -3881.16406  15277.96093

**********

REQUIRED MOMENT APPLIED= 203.68218  19984.07812

POWER FOR ALPHA= 229.42288  1770.43554

RATE MAG-ARG= 0.08933  -0.11868 MOMENT MAG-ARG= 19985.10546  1.56060

AVERAGE POWER REQUIRED FOR ALPHA= -96.64401 DUE TO POWER FACTOR= -0.10827

**********

REQUIRED MOMENT APPLIED= 3882.28540  -15281.94921

POWER FOR BETA= 1406.53710  -2440.66772

RATE MAG-ARG= 0.17866  0.27402 MOMENT MAG-ARG= 15767.36328  -1.32201

AVERAGE POWER REQUIRED FOR BETA= -35.54822 DUE TO POWER FACTOR= -0.02524
```

```
C********** FOILDYN PLOTTING MIKE TERRY
      COMPLEX CCM1,CCM2,CCM3,CCM4,CCM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1      ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
     2      MOMTBB,LIFTOT,MOMTOT,MOMTBT,POWERA,POWERB,MOMAPA,MOMAPP,LIF
      DIMENSION D1(54),D2(54),D3(54),D5(54),D6(54),D7(54),D4(54)
      COMMON  RO,B,A,C,MODE,BA,DPHE,PI,G,HW,WL,L,SI,PHEW,C1,C2,C3,C4,
     1      C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2      BDOT,BDDOT,C11
      CALL NEWPLT('M7095','7458','VELLUM','BLACK')
      CALL PLOT1(25.,5.,-3)
C**********READ IN NUMBER OF FOIL RUNS
      READ(5,102)  NPLOT
      READ (5,102)  NMAX
      M=1
C********READ IN FOIL INFORMATION  RO = SLUGS PER FT3, B = 1/2 CHORD IN
C********FT, A = LIFT POINT FROM MIDCHORD AFT IN RATIO OF B, C = FLAP
C********POINT FROM MIDCHORD AFT IN RATIO OF B, MODE = 1 FOR ALPHA ONLY
C********2 FOR BETA ONLY, 3 FOR ALPHA& BETA, BA = RATIO OFMAGBETA TO MAG
C********ALPHA, DPHE = ANGLE BETWEEN BETA & ALPHA IN RADIANS
C**********FIA =FOIL INERTIA IN SLUGS . FT2/FT SPAN
C**********FIB=CONTROL SURFACE INERTIA IN SLUGS FT2/FT SPAN
 5050 WRITE (6,118)  M
  118 FORMAT (1H1,14HFOIL NUMBER = ,I12)
      READ (5,101) RO,B,A,C,MODE,BA,DPHE
  101 FORMAT(4F10.5,I12,2F10.5)
      READ (5,113)  FIA,FIB
  113 FORMAT(2F20.5)
C********SET UP FOR MULTIPLE WAVE RUNS NMAX
      READ (5,102) NMAX
  102 FORMAT (I12)
      N = 1
      PI = 3.1416
      G = 32.2
C********IF C = 0 OR NOT CALCULATE CONSTANTS C1 TO C10
      IF(C.EQ.0.0) GO TO 200
      CCC = 1.0 - C**2
      SRTC = SQRT(CCC)
```

```
      ACC = ATAN2(SRTC,C)
      GO TO 300
  200 CCC = 1.0
      SRTC = 1.0
      ACC = PI/2.0
  300 C1 = C*SRTC - ACC
      C2 = C*ACC - .332*(2.0 + C**2)
      C3 =    SRTC + ACC
      C4 = (1.0 - 2.0*C)*ACC + (2.0 - C)*SRTC
      C5 = .332*SRTC*(C**2 - 1.0)  -  (C - A)*C1
      C6 = (0.125 + C**2)*ACC - 0.125*C*SRTC*(7.0 + 2.0*C**2) + (C - A)
     1     *(-C2)
      C7 = A*C1 + 0.332*SRTC**3 + C2
      C8 = 2.0*C*SRTC*ACC - CCC - ACC**2
      C9 = 0.25*C*SRTC*ACC*(7.0 + 2.0*C**2) - (0.125 + C**2)*ACC**2
     1     -0.125*CCC*(5.0*C**2 + 4.0)
      C10 = (1.0 + 0.5*C)*SRTC - (C + 0.5)*ACC
      C11 = (1.0  -  0.5*C)*SRTC    +    (0.5  -  C)*ACC
      WRITE (6,301) C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11
  301 FORMAT (11F10.5)
C********READ IN WAVE INFORMATION HW = WAVE HALF HEIGHT IN FT, WL = WAVE
C********LENGTH IN FT, U = SHIP VELOCITY IN FT/SEC, SI = ANGLE BETWEEN U
C********AND ONCOMING WAVE PHASE VELOCITY IN RADIANS, PHEW = REF WAVE
C*********ORBITAL UPWASH VELOCITY PHASE ANGLE IN RADIANS
 400  READ (5,103) HW,WL,U,SI,PHEW
  103 FORMAT (5F10.5)
      WRITE (6,108)  N
  108 FORMAT (1H1,13HRUN NUMBER = ,I12,9HFOIL DATA)
      WRITE (6,109)  RO,B,A,C,MODE,BA,DPHE,FIA,FIB
  109 FORMAT (1H0,10HDENSITY = ,1F10.5,12HSEMICHORD = ,F10.5,
     1  13HLIFT POINT = ,F10.5,13HFLAP POINT = ,F10.5,/,1H0,
     2     14HCONTRL MODE = ,I12,16HBETA TO ALPHA = ,F10.5,
     3     22HBETA TO ALPHA ANGLE = ,F10.5,15HFOIL INERTIA = ,
     4     F20.5,/,1H0,25HCONTRL SURFACE INERTIA = ,F20.5)
      WRITE ( 6,110) HW,WL,U,SI,PHEW
  110 FORMAT(1H0,9HWAVE DATA,/,1H0,14HWAVE HEIGHT = ,F10.5,
     1  14HWAVE LENGTH = ,F10.5,16HVEHICLE SPEED = ,F10.5,/,1H0,
```

```
      2  17HWAVE CIRECTION = ,F10.5,15HWAVE PHASE ANGLE = ,F1C.5)
C**********CALCULATE W2=WAVE FREQ **2 IN RAD/SEC
      W2 = (2C2.C)/WL
C**********CALCULATE ENCOUNTER FREQ IN RAD/SEC FROM WL, U, SI
      WE = SQRT(W2) + (W2*U*CCS(SI))/G
       C1(N) = WE/.75
      IF(U.EQ.0.0)  GC TC 900
C**********CALCULATE CA2=REDUCED FREQUENCY BASED CN SEMICHCRD
      CA2 = WE*B/U
      CA = ABS(CA2)
      CALL BESJ(CA,0,BCK,C.C1,IER)
      CALL BESJ(CA,1,B1K,C.C1,IER)
C**********TEST FCR SMALL CA BEFORE TRYING BESY
      IF(CA - C.C5 )  6CC, 6CC, 7CC
  600 CK = (1.0,-0.000005)
      GO TC 8CC
  900 CK = (C.5,-C.C000025)
      GC TC 800
  7C0 CALL BESY(CA,1,Y1K,IER)
      CALL BESY(CA,0,YCK,IER)
      CCM1 = CMPLX(B1K,-Y1K)
      COM2 = CMPLX(BCK,-YCK)
      CCM3 = (C.C,1.0)
      CCM4 = CCM3*CCM2
      CK = COM1/(COM1 + COM4)
  800 WRITE(6,104) WE,CK,CA2
  104 FORMAT (1H0,17HENCCUNTER FREQ = ,F10.5,17HTHEODORSEN FUN.= ,
     1      2F10.5,15HREDUCED FREQ = ,F1C.5)
C*********CALCULATE LIFT CUE TC WAVES
 10C0 CALL WAVLIF(W2,CA,       MCMTW,MCMTBW)
C*********SCLVE FCR ALPHA & BETA
 20C0 CALL CCNTFL(ALPH ,BET )
      IF(MCDE.EQ.1) GO TO 3CCC
      IF(MCDE.EQ.2) GO TO 4000
C*********CALCULATE LIFT DUE TC ALPHA
 30C0 CALL LIFA(LIFTA,MOMTA,MOMTBA)
      IF(MCDE.EQ.1) GC TO 5000
```

```
C*********CALCULATE LIFT DUE TO BETA
 4000 CALL LIFB(LIFTB,MOMTB,MOMTBB)
 5000 IF(MODE.EQ.1) GO TO 7000
      IF(MODE.EQ.2) GO TO 8000
      GO TO 9000
C*********INITIALIZE ALPHA & BETA
 7000 LIFTB = (0.0,0.0)
      MOMTB = (0.0,0.0)
      MOMTBB = (0.0,0.0)
      GO TO 9000
 8000 LIFTA = (0.0,0.0)
      MOMTA = (0.0,0.0)
      MOMTBA = (0.0,0.0)
C********* TOTALS
 9000 LIFTOT = LIFTW + LIFTA + LIFTB
      MOMTOT = MOMTW + MOMTA + MOMTB
      MOMTBT = MOMTBW + MOMTBA + MOMTBB
      IF(MODE.EQ.1) GO TO 9002
      IF(MODE.EQ.2) GO TO 9003
C**********MOMENT APPLIED EQUALS I*ADDOT MINUS SUM HYD MOMENT
 9002 MOMAPA = FIA*ADDOT  - MOMTOT
      POWERA = ADOT*MOMAPA
      CALL FTAN(ADOT,ADMAG,PHEAD)
      CALL FTAN(MOMAPA,PMAMAG,PHEMPA)
      WRITE(6,117)  ADMAG,PHEAD,PMAMAG,PHEMPA
  117 FORMAT(1H0,15HRATE MAG-ARG = ,2F10.5,17HMOMENT MAG-ARG = ,2F20.10)
C**********FINDING MAG & ANGLE FOR AVERAGE POWER CALC
      PHEPA = PHEAD - PHEMPA
      PFACTA = COS(PHEPA)
      PAVEA = 0.5*ADMAG*PMAMAG*PFACTA
      WRITE (6,115)  PAVEA,PFACTA
  115 FORMAT(1H0,35HAVERAGE POWER REQUIRED FOR ALPHA = ,F20.5,
     1     22HDUE TO POWER FACTOR = ,F10.5)
      CALL FTAN(ALPHA,AMAG,PHEA)
      D2(N) = AMAG/.394
      D3(N) = PHEA/.788
       D4(N) = PAVEA/400.
```

```
      IF(MODE.EQ.1) GO TO 9001
 9003 MOMAPB = FIB*BDDOT  -  MOMTBT
      POWERB = BDOT*MOMAPB
      CALL FTAN(BDOT,BDMAG,PHEBD)
      CALL FTAN(MOMAPB,PMBMAG,PHEMPB)
      WRITE(6,117)  BDMAG,PHEBD,PMBMAG,PHEMPB
      PHEPB = PHEBD  -  PHEMPB
      PFACTB = COS(PHEPB)
      PAVEB = 0.5*BDMAG*PMBMAG*PFACTB
      WRITE(6,116) PAVEB,PFACTB
  116 FORMAT(1H0,34HAVERAGE POWER REQUIRED FOR BETA = ,F20.5,
     1      22HDUE TO POWER FACTOR = ,F10.5)
      CALL FTAN(BETA,BMAG,PHEB)
      D5(N) = BMAG/.394
      D6(N) = PHEB/.788
      D7(N) = PAVEB/400.
      IF(MODE.EQ.2) GO TO 9001
      D7(N) = -D7(N)
 9001 N = N + 1
      IF(N-NMAX) 400,400,6000
 6000 X1 = D1(1)
      Y2 = D2(1)
      Y3 = D3(1)
      Y4 = D4(1)
      Y5 = D5(1)
      Y6 = D6(1)
      Y7 = D7(1)
      X11 = D1(NMAX)
      Y22 = D2(NMAX)
      Y33 = D3(NMAX)
      Y44 = D4(NMAX)
      Y55 = D5(NMAX)
      Y66 = D6(NMAX)
      Y77 = D7(NMAX)
      IF(MODE.EQ.1) GO TO 6010
      IF(MODE.EQ.2) GO TO 6020
 6010 DO   6011  I = 1, NPLOT
```

```
      CALL AXIS1(-10.,-4.,'ALPHA MAG = A',13,3.,90.,0.00,22.5,1,0,1.)
      CALL AXIS1(-8.75,-4.,'ALPHA ANG = Z',13,3.,90.,0.000,45.,1,0,1.)
      CALL AXIS1(-7.5,-4.0,'APWR PG FTLB/SEC',16,3.0,90.,0.0000,400.,0,
     1        0,1.)
      CALL AXIS1(.0,-4.0,' ',1,4.0,90.,1.,.0,0,0,1.)
      CALL AXIS1(-7.0,.0,'WE RAD/SEC',10,7.0,.0,0.00,.75,2,0,1.)
      CALL SYMBL5(X1    ,Y2    ,.25,193,.0,-1)
      CALL GRAPH(D1,D2,NMAX,0,0)
      CALL SYMBL5(X11       ,Y22       ,.25,193,.0,-1)
      CALL SYMBL5(X1    ,Y3    ,.25,233,.0,-1)
      CALL GRAPH(D1,D3,NMAX,0,0)
      CALL SYMBL5(X11       ,Y33       ,.25,233,.0,-1)
      CALL SYMBL5(X1    ,Y4    ,.25,215,.0,-1)
      CALL GRAPH(D1,D4,NMAX,0,0)
      CALL SYMBL5(X11       ,Y44       ,.25,215,.0,-1)
      IF(MODE.EQ.1) GO TO 6012
      CALL SYMBL5(X1    ,Y7    ,.25,216,.0,-1)
      CALL GRAPH(D1,D7,NMAX,0,0)
      CALL SYMBL5(X11       ,Y77       ,.25,216,.0,-1)
 6012 CALL PLOT1(25.,.0,-3)
 6011 CONTINUE
      GO TO 6040
 6020 DO  6021   I = 1,NPLOT
      CALL AXIS1(-10.,-4.,'BETA  MAG = B',13,3.,90.,0.00,22.5,1,0,1.)
      CALL AXIS1(-8.75,-4.,'BETA  ANG = Y',13,3.,90.,0.000,45.,1,0,1.)
      CALL AXIS1(-7.5,-4.0,'APWR PW FTLB/SEC',16,3.0,90.,0.0000,400.,0,
     1      0,1.)
      CALL AXIS1(.0,-4.0,' ',1,4.0,90.,1.,.0,0,0,1.)
      CALL AXIS1(-7.0,.0,'WE RAD/SEC',10,7.0,.0,0.00,.75,2,0,1.)
      CALL SYMBL5(X1    ,Y5    ,.25,194,.0,-1)
      CALL GRAPH(D1,D5,NMAX,0,0)
      CALL SYMBL5(X11       ,Y55       ,.25,194,.0,-1)
      CALL SYMBL5(X1    ,Y6    ,.25,232,.0,-1)
      CALL GRAPH(D1,D6,NMAX,0,0)
      CALL SYMBL5(X11       ,Y66       ,.25,232,.0,-1)
      CALL SYMBL5(X1    ,Y7    ,.25,216,.0,-1)
      CALL GRAPH(D1,D7,NMAX,0,0)
```

```
      CALL SYMBL5(X11      ,Y77      ,.25,216,.C,-1)
 6022 CALL PLCT1(25.,.C,-3)
 6C21 CCNTINUE
      GO TC 6C4C
 6040 N = N + 1
      IF(N-NMAX)  505C,5050,5080
 5080 CALL ENCPLT
      CALL EXIT
      END
      SUBRCUTINE WAVLIF(W2,CA,        MOMTW,MOMTBW)
      CCMPLEX CCM1,CCM2,CCM3,CCM4,COM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1      ADOT,ADDCT,BETA,BDCT,BDDOT,LIFTA,MCMTA,MCMTBA,LIFTB,MCMTB,
     2      MCMTBB,LIFTOT,MOMTOT,MOMTBT,POWERA,PCWERB,MCMAPA,MCMAPB,LIF
      CCMMCN  FC,B,A,C,MCDE,BA,CPHE,PI,G,HW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1      C5,C6,C7,C8,C9,C1C,WE,CK,LIFTW,ALPHA,BETA,ACCT,ADDCT,
     2      BDCT,BDDOT,C11
C*********PEAK VELOCITY  UPWASH
      UW = HW*SQRT(W2)
      CUW = UW*CCS(PHEW)
      SUW = UW*SIN(PHEW)
      UPW = CMPLX(CUW,SUW)
      CCM3 = (C.C,1.C)
      CALL BESJ(CA,C,BCK,C.C1,IER)
      CALL BESJ(CA,1,B1K,0.01,IER)
      CCM5 = CMPLX(BOK,-B1K)
      LIF = CK*COM5   +  COM3*B1K
C*********CHK FCR + _WE
      IF(WE - C.C)  500,50C,6CC
  600 LIFTW = 2.0*PI*RO*U*B*UPW*CONJG(LIF)
      GC TC 700
  500 LIFTW = 2.C*PI*RO*U*B*UPW*LIF
C*********MOMENT ABOUT Y
  700 MCMTW = B*(C.5  +  A)*LIFTW
C*********MOMENT ABOUT C
      MCMTBW =  B*(-C11)*LIFTW/PI
      RETURN
      END
```

```
      SUBROUTINE CONTRL(ALPH ,BET )
      COMPLEX CCM1,CCM2,CCM3,CCM4,CCM5,CK,UPW,LIFTW,MCMTW,MCMTBW,ALPHA,
     1        ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MCMTA,MCMTBA,LIFTB,MCMTB,
     2        MCMTBB,LIFTCT,MCMTCT,MCMTBT,POWERA,POWERB,MOMAPA,MOMAPB,LIF
      COMMON  RO,B,A,C,MODE,BA,DPHE,PI,G,HW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1        C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2        BDOT,BDDOT,C11
C**********CALCULATE CONSTANTS
C*********MAKE +DPHE LEADING
      IF(WE - 0.0) 2000,2000,3000
 3000 XPHE = -DPHE
      GO TO 4000
 2000 XPHE = DPHE
 4000 CDPHE = COS(XPHE)
      SDPHE = SIN(XPHE)
      C1CP = C1*CDPHE
      C2CP = C2*CDPHE
      C3CP = C3*CDPHE
      C4CP = C4*CDPHE
      C1SP = C1*SDPHE
      C2SP = C2*SDPHE
      C3SP = C3*SDPHE
      C4SP = C4*SDPHE
      CKR = REAL(CK)
C**********CHK FOR + _ WE
      IF(WE - 0.0)  900,900,1000
 1000 CKI = -AIMAG(CK)
      GO TO 1100
  900 CKI = AIMAG(CK)
 1100 IF(MODE.EQ.1) GO TO 100
      IF(MODE.EQ.2) GO TO 200
      IF(MODE.EQ.3) GO TO 300
      GO TO 600
  100 WRITE(6,101)
  101 FORMAT(1H0,10H**********,/,1H0,18HMODE IS ALPHA ONLY)
      GO TO 400
  200 WRITE (6,102)
```

```
  102 FORMAT(1H0,10H**********,/,1H0,17HMODE IS BETA ONLY)
C**********BETA ONLY EQS
      Y1 = -B*WE*(U*C1CP +B*WE*C2SP) -2.0*U*(U*CKR*C3SP + U*CKI*C3CP -B*
     10.5*WE*CKR*C4CP + B*0.5*WE*CKI*C4SP)
      Y2 = -B*WE*(U*C1SP - B*WE*C2CP)  +2.0*U*(U*CKR*C3CP -U*CKI*C3SP +
     1 B*0.5*WE*CKR*C4SP +B*0.5*WE*CKI*C4CP)
      X1 = -B*WE*(U*C1SP -B*WE*C2CP) +2.0*U*(-U*CKI*C3SP +U*CKR*C3CP +
     1 B*0.5*WE*CKI*C4CP  +B*0.5*WE*CKR*C4SP)
      X2 = -B*WE*(-U*C1CP -B*WE*C2SP)  +2.0*U*(U*CKI*C3CP + U*CKR*C3SP+
     1 B*0.5*WE*CKI*C4SP  -B*0.5*WE*CKR*C4CP)
      GO TO 500
  300 WRITE (6,103) BA,DPHE
  103 FORMAT(1H0,10H**********,/,1H0,
     1      33HMODE IS ALPHA AND BETA WITH BA = ,F10.5,12H AND DPHE = ,
     2      F10.5)
C**********ALPHA ONLY OR ALPHA + BETA EQS
  400 Y1 = PI*B*WE*U  -B*BA*WE*(U*C1CP  +B*WE*C2SP)  -2.0*PI*U*(U*CKI -
     1 B*(0.5 -A)*WE*CKR)   -2.0*U*BA*(U*CKR*C3SP + U*CKI*C3CP -
     2 B*0.5*WE*CKR*C4CP + B*0.5*WE*CKI*C4SP)
      Y2 = PI*B*WE*B*A*WE  -B*BA*WE*(U*C1SP  -B*WE*C2CP)  +2.0*PI*U*(U*
     1CKR +B*WE*CKI*(0.5 -A)) +2.0*U*BA*(U*CKR*C3CP -U*CKI*C3SP +
     2B*0.5*WE*CKR*C4SP +B*0.5*WE*CKI*C4CP)
      X1 = PI*B**2*WE**2*A  -B*BA*WE*(U*C1SP  -B*WE*C2CP) +2.0*PI*U*(U*
     1CKR + B*(0.5 -A)*WE*CKI) +2.0*U*BA*(-U*CKI*C3SP +U*CKR*C3CP +
     2B*0.5*WE*CKI*C4CP  +B*0.5*WE*CKR*C4SP)
      X2 = -PI*B*WE*U  -B*BA*WE*(-U*C1CP  -B*WE*C2SP)  +2.0*PI*U*(U*
     1CKI  -B*(0.5 -A)*WE*CKR)  +2.0*U*BA*(U*CKI*C3CP +U*CKR*C3SP +
     2  B*0.5*WE*CKI*C4SP  -B*0.5*WE*CKR*C4CP)
  500 ALR = REAL(LIFTW)
      ALI = AIMAG(LIFTW)
      R = ALR/ALI
      Y = Y2  -  R*X2
      X = R*X1  -  Y1
C**********SOLVE FOR ANGLE
      PHEA = ATAN2(Y,X)
      CPHEA = COS(PHEA)
      AAA = Y1*Y/X  +  Y2
```

```
C**********SOLVE FOR MAGNITUDE
      ALPH = -ALR/(CPHEA*RO*B*AAA)
      ALPHA = CMPLX(ALPH*COS(PHEA),ALPH*SIN(PHEA))
      BET = BA*ALPH
      PHEB = PHEA + XPHE
      BETA = CMPLX(BET*COS(PHEB),BET*SIN(PHEB))
      IF(MODE.EQ.1) GO TO 700
      IF(MODE.EQ.2) GO TO 800
  700 WRITE(6,104) ALPHA
  104 FORMAT(1H0,18HALPHA REAL-IMAG = ,2F10.5)
      COM3 = (0.0,1.0)
C**********SOLVE FOR ALPHA DOT AND ALPHA DOUBLE DOT
      ADOT = -COM3*WE*ALPHA
      ADDOT = -WE**2*ALPHA
      IF(MODE.EQ.1) GO TO 600
  800 WRITE(6,105) BETA
  105 FORMAT(1H0,17HBETA REAL-IMAG = ,2F10.5)
      COM3 = (0.0,1.0)
C**********SOLVE FOR BETA DOT AND BETA DOUBLE DOT
      BDOT = -COM3*WE*BETA
      BDDOT = -WE**2*BETA
  600 CONTINUE
      RETURN
      END
      SUBROUTINE LIFA(LIFTA,MOMTA,MOMTBA)
      COMPLEX COM1,COM2,COM3,COM4,COM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1     ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
     2     MOMTBB,LIFTOT,MOMTOT,MOMTBT,POWERA,POWERB,MOMAPA,MOMAPB,LIF
      COMMON  RO,B,A,C,MODE,BA,DPHE,PI,G,HW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1     C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2     BDOT,BDDOT,C11
C**********TEST FOR + _ WE
      IF(WE - 0.0) 500,500,600
  600 LIF = CONJG(CK)
      GO TO 700
  500 LIF = CK
  700 LIFTA = PI*RO*B**2*(U*ADOT -B*A*ADDOT)  +  2.0*PI*RO*U*B*
```

```
1      (U*ALPHA  +  B*(0.5 - A)*ADOT)*LIF
C********** MOMENT ABOUT Y
      MOMTA = PI*RO*B**2*(U**2*ALPHA  - B**2*(0.125 + A**2)*ADDOT)
     1      -2.0*PI*RO*U*B**2*(U*ALPHA + B*(0.5 - A)*ADOT)*(0.5
     2      -(A + 0.5)*LIF)
C**********MOMENT ABOUT C
      MOMTBA =-RO*B**2*U**2*C1*ALPHA +RO*B**3*U*C7*ADOT-RO*B**4*C6*ADDOT
     1      -2.0*RO*U*B**2*(U*ALPHA + B*(0.5 -A)*ADOT)*(-0.5*C1 + C10*
     2      LIF)
      RETURN
      END
      SUBROUTINE LIFB(LIFTB,MOMTB,MOMTBB)
      COMPLEX COM1,COM2,COM3,COM4,COM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1      ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
     2      MOMTBB,LIFTOT,MOMTOT,MOMTBT,POWERA,POWERB,MOMAPA,MOMAPB,LIF
      COMMON  RO,B,A,C,MODE,BA,DPHE,PI,G,HW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1      C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2      BDOT,BDDOT,C11
C**********TEST FOR + _ WE
      IF(WE - 0.0) 500,500,600
  600 LIF = CONJG(CK)
      GO TO 700
  500 LIF = CK
  700 LIFTB = -RO*B**2*U*C1*BDOT-RO*B**3*C2*BDDOT + RO*2.0*U*B*
     1      (U*C3*BETA + 0.5*B*C4*BDOT)*LIF
C**********MOMENT ABOUT Y
      MOMTB = -RO*B**2*U**2*C1*BETA -RO*B**3*U*C5*BDOT -RO*B**4*C6*BDDOT
     1      -2.0*RO*U*B**2*(U*C3*BETA + 0.5*B*C4*BDOT)*(0.5 -(A+0.5)*LIF)
C**********MOMENT ABOUT C
      MOMTBB = -RO*B**2*U**2*C8*BETA/PI +RO*B**4*C9*BDDOT/PI
     1      -2.0*RO*U*B**2*(U*C3*BETA/PI +0.5*B*C4*BDOT/PI)*(-0.5*C1 +
     2      C10*LIF)
      RETURN
      END
      SUBROUTINE FTAN(ARG,AMAG,PHE)
      COMPLEX COM1,COM2,COM3,COM4,COM5,CK,UPW,LIFTW,MOMTW,MOMTBW,ALPHA,
     1      ADOT,ADDOT,BETA,BDOT,BDDOT,LIFTA,MOMTA,MOMTBA,LIFTB,MOMTB,
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```
     2      MCMTBB,LIFTCT,MCMTCT,MOMTBT,PCWERA,PCWERB,MOMAPA,MOMAPB,ARG,
     3      LIF
      COMMON  RC,E,A,C,MODE,BA,DPHE,PI,G,HW,WL,U,SI,PHEW,C1,C2,C3,C4,
     1      C5,C6,C7,C8,C9,C10,WE,CK,LIFTW,ALPHA,BETA,ADOT,ADDOT,
     2      BDOT,BDDOT,C11
C**********FINDS MAG AND ANGLE OF ARG
      X = REAL(ARG)
      Y = AIMAG(ARG)
      IF(X.EQ.0.0) GO TO 200
      PHE = ATAN2(Y,X)
      GO TO 300
  200 IF(Y - 0.0) 400,500,600
  400 PHE = -PI/2.0
      GO TO 300
  500 PHE = PI/4.0
      GO TO 300
  600 PHE = PI/2.0
  300 AMAG = CABS(ARG)
      RETURN
      END
```

```
RUN NUMBER= 1 FOIL DATA

DENSITY= 2.0 SEMICHORD= 4.29000 LIFT POINT= 0.0 FLAP POINT = 0.50000

CONTRL MODE= 3 BETA TO ALPHA 2.0 BETA TO ALPHA ANGLE= -0.39270 FOIL INERTIA= 140.00

CONTRL SURFACE INERTIA= 8.78000

WAVE DATA

WAVE HEIGHT= 7.00000 WAVE LENGTH= 280.00000 VEHICLE SPEED= 80.00000

WAVE DIRECTION= 0.0 WAVE PHASE ANGLE= -1.57080

ENCOUNTER FREQ= 2.64174 THEODORSEN FUN.= 0.78157  -0.18519 REDUCED FREQ= 0.14166

**********

MODE IS ALPHA AND BETA WITH BA= 2.0 AND DPHE= -0.39270

ALPHA REAL-IMAG= 0.00400  0.03358

BETA REAL-IMAG= -0.01830  0.06511

RATE MAG-ARG= 0.08933  -0.11868 MOMENT MAG-ARG= 19985.10546  1.56060

AVERAGE POWER REQUIRED FOR ALPHA= -96.64401 DUE TO POWER FACTOR= -0.10827

RATE MAG-ARG= 0.17866  0.27402 MOMENT MAG-ARG= 15767.36328  -1.32201

AVERAGE POWER REQUIRED FOR BETA= -35.54822 DUE TO POWER FACTOR= -0.02524
```